Corporate Taxation in the Open Economy without Pareto

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Abstract

This paper studies how optimal corporate tax rates differ when firms draw their productivity from a log-normal distribution, which better fits the data, instead of a Pareto distribution, the literature standard, in a model with heterogeneous sectors and monopolistic competition. Using an enhanced multi-sector Melitz model with corporate taxation, I find that the distributional choice has significant implications for the properties of the optimal corporate tax rates. The corporate tax framework consists of a single economy-wide statutory tax that is augmented by a set of sector-specific depreciation allowance rates that distort the effective tax rate of each sector. I find that using the Pareto distribution mutes a transmission channel between the corporate tax instruments and the equilibrium variables, which leads to qualitative different policy implications compared to those obtained under the log-normal distribution. Additionally, my model can reconcile recent empirical studies that come to seemingly conflicting conclusions about the effects of statutory tax rates on export dynamics. I show that the level of the sector-specific depreciation allowance determines whether or not changing a decrease in the statutory corporate tax rate increases the export probability of firms.

Keywords: Corporate tax policy, Melitz-Pareto, asymmetric sectors.

JEL Classification Numbers: F12, F68, H25.

1. Introduction

The trade literature with heterogeneous firms has mostly assumed that firm productivity follows a Pareto distribution.[1] Recent studies have started a debate on how this "standard" assumption affects the outcomes of the models in question, with particular attention to the most widely used model of this type: the Melitz model. For example, Head et al. (2014) find implications for the size of the gains from trade (GFT) and also

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[1] The justification for this assumption has roots in empirical evidence from Axtell (2001), Del Gatto et al. (2006). However, the real advantage of using the Pareto distribution lyes in the analytical tractability that it provides to the models.

show that a model using log-normal distributions provides a significantly better fit to firms' sales data from France and Spain. Furthermore, Bee and Schiavo (2018) provide a thorough comparison between the GFT obtained under both distributions to highlight that the standard assumption might be overstating the gains of trade in a significant way. I follow in these steps, but on a parallel path, by investigating the implications to optimal corporate taxation in a Melitz model when one departs from the standard assumption of Pareto distributions for productivity in favor of a log-normal distribution. I also provide evidence that the latter distribution is consistently a better fit for productivities in over 100 countries that are part of the World Bank Entrepreneurial Survey.

This paper studies a multi-sector trade model à la Melitz, in which I include governments that must provide a fixed amount of public goods financed through the taxation of firms' profits. The corporate tax framework aims to capture the system observed in most countries. This tax system consists of a single statutory corporate profit tax rate (τ), imposed on all firms producing in the country; and a set of sector-specific capital depreciation allowance rates (δ_s), which in the case of my model is assessed on the fixed cost of production. The particularity of this corporate tax framework is that the *effective tax rate* is not only different from the statutory tax rate, but it can vary across sectors.[2] Through this paper, I refer to the set of τ and δs as corporate tax rates or fiscal instruments.

The question of what are the optimal corporate tax rates has substantially different answers depending on the assumed productivity distribution. For example, the depreciation allowance rates (δ), under the assumption of Pareto distributions, do not explicitly include sector-specific fixed costs of production or entry. On the other hand, the optimal policy for the government in the log-normal model is to exploit all of these sector cost asymmetries by using a targeted approach through δ instead of τ which has an economy-wide scope. In the context of an open economy model with symmetric countries, the probability of exporting is invariant to changes in tax rates when assuming Pareto distributions. This "export neutrality" translates into optimal corporate tax rates that do not change when a country moves from autarky to trade. Such property fails to hold in the log-normal case as the government must adjust their optimal corporate tax rates since their power to influence the equilibrium outcomes decreases when the country opens to trade.

When assuming Pareto distributions, a channel of transmission shuts down, thereby generating the differences in the optimal formulas for the fiscal instruments. This channel consists of the productivity ratio between the average and marginal firm; this ratio is constant under Pareto but variable under the log-normal distribution. Therefore, assuming Pareto distributions eliminates one channel through which governments

[2] Effective tax rates are the ratio of taxes paid over net profits. For a recent study in the variability of this measure across sector see Barrios et al. (2014).

can influence, via the fiscal instruments, the equilibrium outcomes.

There are non-trivial welfare losses associated with using the simpler policy functions derived under the Pareto assumption in an economy whose productivities follow a log-normal distribution as suggested by the data. In the closed economy, the welfare losses increase with the degree of asymmetry across the sectors, with one of the numerical examples showing a 3% welfare loss relative to using the "correct" policy functions. In the open economy setting, not only does the degree of asymmetry between sectors within a country plays a role, but a more important driver is the heterogeneity between countries. In this setting, the same scenarios considered in the closed economy yield welfare losses 5 to 10 times as high. In the context of policymaking, the significant welfare losses warrant the use of the more complicated functions for the optimal corporate tax rates when appropriate.

Adding the proposed tax framework to a Melitz model also provides a basis to reconcile contradictory findings regarding the relationship between corporate taxes and export status. Bernini and Treibich (2016) find that French small and medium size firms are less likely to export their products when they face higher corporate tax rates. On the other hand, Federici and Parisi (2014) use longitudinal data from Italian firms to find the opposite relation. My model can produce both relationships, and it shows that the export cutoffs are not solely functions of domestic taxes but also depend on taxes in the export target country.

In the model, the tax collected by the government is used to purchase an exogenous amount of a public good q_0^G sold under perfect competition. Thus, governments choose tax rates to maximize the welfare of their citizens while raising enough tax revenue to cover an exogenous level of expenditure. This simple framework can be used to replace the decentralization scheme proposed by Nocco et al. (2014) – to achieve the efficient outcome in a multi-sector Melitz type model – which is based on subsidies and lump-sum transfers.[3] If the amount q_0^G is set to the optimal amount found by Nocco et al., then my model provides a framework to compute the optimal tax rates that could be implemented in current tax codes to achieve such outcome.

[3]Recent papers show that market outcomes are inefficient when the economy is composed of a perfectly competitive sector and a monopolistic competitive one. In particular, Dhingra and Morrow (2012) show that resources are misallocated between such sectors in a Melitz type model with Variable Elasticity of Substitution preferences (see Zhelobodko et al. (2012) for VES preferences exposition) leading to inefficient outcomes that could be improved. Additionally, Nocco et al. (2014) propose a decentralization scheme to achieve an efficient outcome via subsidies and lump-sum taxes on consumers and firms. While this scheme provides us with useful insights into the mechanics at play, it is hard to imagine its applicability in the real world given the amount of information that the central authority would need but most importantly, the tax codes of most countries would have to be scratched entirely. The scheme seems like an impossible task from a practical perspective; therefore, I chose to frame the corporate taxes in the model in a way that is closely related to what we observe in most countries.

2. Closed Model

This section presents an extended Melitz model with asymmetric sectors and the addition of a set of fiscal instruments: a statutory corporate tax rate and a set of sector-specific depreciation allowance rates.[4] The model is first developed in a closed environment as it facilitates the discussion of the relations between the fiscal instruments and the equilibrium outcomes, especially sector productivity and the number of firms producing in each sector. Special focus is put on the consequences that assuming Pareto distributions exert on the response of these variables to changes in the fiscal instruments. The following paragraphs define the model and its equilibrium.

Households

The country is home to L households who inelastically supply one unit of labor to fulfill the demand from firms. The household receives a wage (w) per unit of labor and spends her income on a continuum of differentiated goods $q(\omega)$. Households also derive utility from consuming a government-provided public good (q_0^G). The functional form of utility is quasilinear. The household maximization problem is:

$$\max_{Q_s} \quad q_0^G + \prod_{s=1}^{S} Q_s^{\alpha_s}$$

where Q_s is the aggregate consumption of sector $s \in \{1, 2, ..., S\}$ goods and $\sum_{s=1}^{S} \alpha_s = 1$.

Let Ω_s represent the collection of available goods in sector s; the consumer problem can be broken into S separate maximization problems given by:

$$Q_s = \max_{q(\omega)} \left[\int_{\omega \in \Omega_s} q(\omega)^{\rho_s} \right]^{1/\rho_s} \tag{2.1}$$

such that

$$\int_{\omega \in \Omega_s} p_s(\omega) q(\omega) \leq Y_s$$

where $Y_s = \alpha_s Y$ due to Cobb-Douglas preferences over sectors. Equation (2.1) is a standard C.E.S utility with elasticity of substitution $\sigma_s = 1/(1 - \rho_s)$. Per Dixit and Stiglitz (1977), the price index $P_s =$

[4]Bauer et al. (2014) provides a similar taxation framework but their model considers only one sector with heterogeneous firms and no fixed production or entry costs.

$\left[\int_{\omega\in\Omega_s} p_s(\omega)^{1-\sigma_s}\right]^{1/1-\sigma_s}$ is used to express quantities demanded:

$$q_s(\omega) = \frac{Y_s p_s(\omega)^{-\sigma_s}}{P_s^{1-\sigma_s}} = Q_s \left[\frac{p_s(\omega)}{P_s}\right]^{-\sigma_s} \tag{2.2}$$

Firms

Firms operate in one of the S sectors of the economy under monopolistic competition and costly entry. After paying the sector-specific entry cost ($F_{e,s}$), a firm randomly draws its productivity (φ) from the random distribution $Z_s(\varphi)$. A firm in sector s and productivity φ requires $l = q/\varphi + f_s$ units of labor to produce q units of output. The fixed cost of production (f_s) is homogeneous across firms in the same sector.

The government sets a statutory corporate profit tax rate (τ), that is common for firms regardless of the sector; and a set of sector-specific depreciation allowance rates (δ_s), which allows firms to deduct $\delta_s w f_s$ from their taxable income. Firms know the values of the fiscal instruments (τ, δ) before they make any decision inclusive of entry into a market.

With the above notation, the formulas for taxes paid (t_s), after tax profits (π_s), and the profit maximizing price for a firm with productivity φ in sector s are:

$$t_s(\varphi) = \tau \left(p_s(\varphi)q_s(\varphi) - w\frac{q_s(\varphi)}{\varphi} - \delta_s w f_s\right) \tag{2.3}$$

$$\pi_s(\varphi) = (1-\tau)\left(p_s(\varphi)q_s(\varphi) - w\frac{q_s(\varphi)}{\varphi} - u_s w f_s\right) \tag{2.4}$$

$$u_s = \frac{1-\delta_s\tau}{1-\tau} \tag{2.5}$$

$$p_s(\varphi) = \left(\frac{\sigma_s}{\sigma_s - 1}\right)\frac{w}{\varphi} \tag{2.6}$$

The variable u_s is the user cost of capital, in the spirit of Hall and Jorgensen (1967), when fixed costs of production (f_s) are interpreted as capital that firms spend in order to produce.[5] The type of model that I use does not distinguish between labor and capital (in a neoclassical way) making the interpretation of δ_s less

[5] An implicit assumption in the above equations is a physical depreciation rate of capital of 100%. However, if the real depreciation rate of capital for sector "s" is d_s, the model solution is the same using the modified user cost of capital:

$$u_s = \frac{d_s - \delta_s\tau}{1-\tau}$$

Furthermore, the solutions to the optimal tax problem remain valid by scaling the depreciation allowance rate and the fixed cost of production by the depreciation rate of capital.

$$\hat{\delta}_s = \frac{\delta_s}{d_s}$$
$$\hat{f}_s = d_s f_s$$

straightforward than a depreciation allowance on capital. Thus in this paper, δ_s is a policy instrument that shifts the effective tax rate of firms in sector s only. Holding τ constant, increasing δ_s reduces taxable income of firms in sector s and, ceteris paribus, their effective tax rates decrease.

2.1. Equilibrium

As is well known, in this type of model, the aggregate variables are functions of the average productivity of firms' that find it profitable to produce:

$$\tilde{\varphi}_s(\varphi_s^*) = \left[\frac{1}{1 - Z_s(\varphi_s^*)} \int_{\varphi_s^*}^{\infty} \varphi^{\sigma_s - 1} z(\varphi_s)\, d\varphi\right]^{1/\sigma_s - 1} \tag{2.7}$$

where φ_s^* is the productivity of the marginal firm in sector s, i.e., the firm that makes zero after-tax profit. Let M_s represent the equilibrium number of producing firms in sector s then:

$$\begin{aligned} P_s &= M_s^{1/1-\sigma_s} p_s(\tilde{\varphi}_s) \\ Q_s &= M_s^{1/\rho_s} q_s(\tilde{\varphi}_s) \\ R_s &= M_s r_s(\tilde{\varphi}_s) \end{aligned} \qquad \begin{aligned} \Pi_s &= M_s \pi_s(\tilde{\varphi}_s) \\ T_s &= M_s t_s(\tilde{\varphi}_s) \end{aligned}$$

where $x_s(\tilde{\varphi}_s)$ is the average value of x in sector s, X_s is the sector aggregate value, and r_s is the firm revenue.

The productivity cutoff (φ_s^*) is found by equating two conditions on average *after tax* profits. The first condition comes from the marginal firm, which makes zero after-tax profit:

$$\bar{\pi}_s = (1 - \delta_s \tau) w f_s \left\{ \left[\frac{\tilde{\varphi}_s(\varphi_s^*)}{\varphi_s^*}\right]^{\sigma_s - 1} - 1 \right\} \tag{ZP}$$

Since the number of potential entrants to the market is unbounded, the average expected value of a firm must equal the cost of entry $F_{e,s}$. Let ψ be the probability that a firm goes out of business, then the free entry condition is:

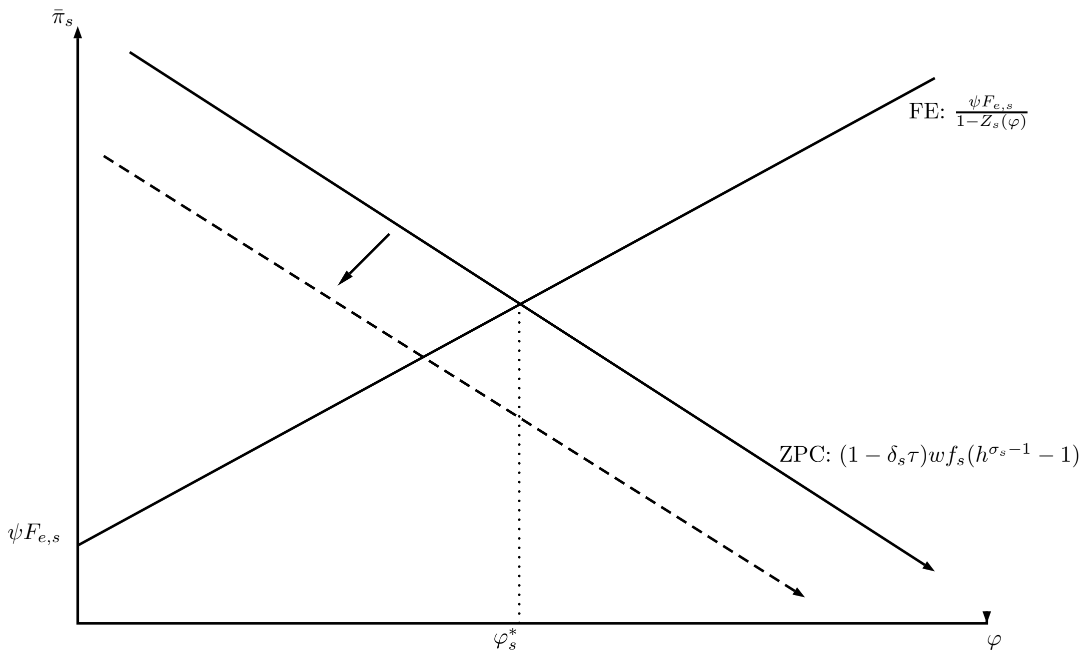
$$\bar{\pi}_s = \frac{\psi}{1 - Z(\varphi_s^*)} w F_{e,s} \quad . \tag{FE}$$

In equilibrium, the (ZP) and (FE) conditions hold in every sector and determine the equilibrium cutoff productivities. Figure 1 shows the graphical representation of the equilibrium φ_s^*.[6]

The last step is to solve for the number of firms in equilibrium by clearing the labor market. The

[6] An equilibrium in which all sectors have a positive number of firms operating only exists if $\delta_s \tau \leq 1$ for all sectors.

Figure 1: Equilibrium productivity cutoff.



economy-wide labor supply L is allocated among firms operating in the S monopolistic competitive sectors and a firm that produces the public good for the government and sells it at marginal cost. A firm with productivity φ has labor costs equal to $r(\varphi) - \pi(\varphi) - t(\varphi)$. Aggregating the expression across firms in sector s results in total labor used for production:

$$wL_{p,s} = R_s - \Pi_s - T_s \qquad \forall s \in S$$

In equilibrium the number of successful new entrants equates the number of exiting firms, thus: $(1 - Z_s(\varphi_s^*))M_{e,s} = \psi M_s$. Using this equality and the FE condition, the labor costs spent in entry ($wL_{e,s}$) equals sector aggregate profit (Π_s). Thus, total labor costs in sector s is:

$$wL_s = wL_{p,s} + wL_{e,s} = R_s - T_s$$

Summing the above across sectors gives the total labor expenditure by firms in the monopolistic competition sectors. Finally, the firm that produces public good uses one unit of labor to produce one unit of q_0^G. Adding the labor used for the production of private consumption goods plus that of the public good results in total labor income:

$$wL = \sum_{s=1}^{S} R_s - \sum_{s=1}^{S} T_s + wq_0^G \tag{2.8}$$

Using the aggregate variable identities defined earlier, the above is transformed into the equations for the equilibrium number of firms:

$$M_{s'} = \frac{\alpha_{s'} \left(wL + \sum_{s=1}^{S} T_s - p_0^G q_0^G \right)}{\sigma_{s'} u_{s'} f_{s'} h_{s'}^{\sigma-1}} \qquad \forall s' \in S \tag{2.9}$$

where $p_0^G = w$ is the price of q_0^G. For the closed economy I use the public good as the numéraire hence $w = 1$.

2.2. Fiscal Instruments and Equilibrium

The following paragraphs describe the relation between equilibrium variables and the fiscal instruments: statutory tax rate (τ) and depreciation allowance rates (δ_s). A set of propositions show the differences between the equilibrium responses under the Pareto and log-normal distributional assumptions for firms' productivity. The source of difference trace to the annihilation of a transmission channel under the standard assumption, i.e., Pareto distributed productivities.

Before proceeding, I define the following variables to facilitate notation and discussion:

$$h_s = \frac{\tilde{\varphi}_s(\varphi_s^*)}{\varphi_s^*} \qquad\qquad \xi_{x,y} = \frac{\partial X}{\partial Y}\frac{Y}{X}$$

where h_s is a measure of firm dispersion and $\xi_{x,y}$ is the elasticity of variable x with respect to variable y.[7]

I start by describing the negative relationship between the depreciation allowance rate and the equilibrium cutoff productivity for the relevant sector. To illustrate, consider an increase in $\delta_{s'}$ which translates into a reduction in the user cost $u_{s'}$, therefore decreasing the after-tax fixed costs of production ($u_{s'} f_{s'}$). The decrease in costs implies that the revenue required to make a zero after-tax profit has decreased; consequently, the productivity cutoff for sector s' falls. In terms of the equilibrium conditions, the increase in $\delta_{s'}$ shifts the ZP curve downward for sector s' since τ is greater than zero as long as there is a positive supply of the public good. In Figure 1, this shift is represented by the dash line which results in a smaller value for $\varphi_{s'}^*$.

Next, I show an ambiguous relationship between τ and the productivity cutoffs, and how the relationship depends on the sign of the depreciation allowance rate for the sector. An important consequence is that changing τ affects all sectors simultaneously, but the direction of change of φ^* can differ across sectors. The

[7] $h^{\sigma-1}$ is the ratio of revenues between the average and marginal firm. An h_s closer to one implies less heterogeneity, in terms of productivities, in sector s. An $h_s = 1$ is equivalent to a model with one representative firm in sector s.

correspondence below shows the sign of the change in φ^* after an increase in τ :

$$\tau \uparrow \begin{cases} \varphi_s^* \downarrow & \text{if } \delta_s > 0 \\ \varphi_s^* \uparrow & \text{if } \delta_s < 0 \\ \varphi_s^* = & \text{if } \delta_s = 0 \end{cases}$$

The above relationships are a direct implication of the $(1 - \delta\tau)$ factor in the ZP equation. Note that a change of $\triangle\tau$ results in a net operating profit change of $(\triangle\tau)\delta w f_s$. When $\delta > 0$, an increase in τ raises net profit, ceteris paribus, thereby reducing the threshold productivity for the marginal firm since making a zero after-tax profit is now "easier". The case with $\delta < 0$ has the exact opposite implication as net profits decrease for any level of productivity.

Now that the links between the tax instruments and the cutoff productivities have been established I show that the change in average productivity has a special property under the Pareto assumption. Clearly, an increase in φ_s^* raises $\tilde{\varphi}_s$, regardless of distribution, but the relation is stronger under Pareto:

Proposition 1 *For any random distribution $Z(\varphi)$ the value of $\xi_{\tilde{\varphi},\varphi^*}$ is strictly positive. If $Z \sim log\mathcal{N}$ then $\xi_{\tilde{\varphi},\varphi^*} < 1$. If the random distribution is Pareto then $\xi_{\tilde{\varphi},\varphi^*} \equiv 1$ across the whole support of φ.*

Proof. Appendix B.1 ■

The property of proposition 1, illustrated in figure 2, is key since changes in τ, δ lead to alterations in h when the distribution is log-normal, while a Pareto distribution implies a constant value of h. In simple terms, assuming Pareto distributed productivities precludes a sector recomposition that results in a wider/narrower disparity between the marginal and average firm. Furthermore, the constant versus variable h has consequences for the equilibrium cutoff productivities, it appears in the ZP equation, and the number of firms, it appears in the denominator.

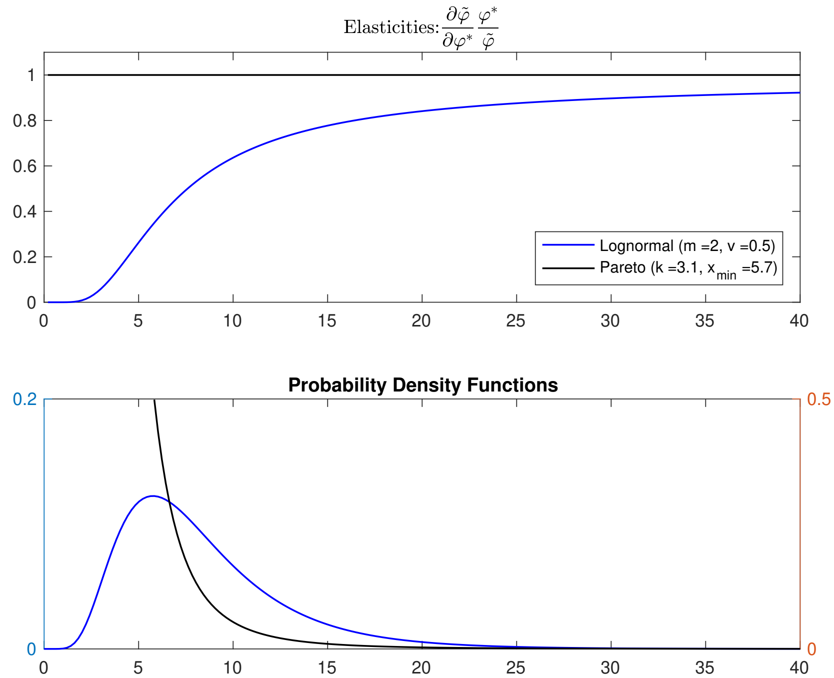
The value $\xi_{\tilde{\varphi},\varphi^*}$ is key to the response of the number of firms to tax rate changes. To illustrate, the elasticity of M_s with respect to statutory tax rate and depreciation allowance rate are:

$$\xi_{M_s,\delta_{s'}} = \frac{\sum_{s=1}^{S} \frac{\partial T_s}{\partial \delta_{s'}} \delta_{s'}}{wL + \sum_{s=1}^{S} T_s - p^g q_0^G} - \left[\frac{-\tau\delta_{s'}}{(1-\delta_{s'}\tau)} + (\sigma_{s'} - 1)\left(\xi_{\varphi_s^*,\delta_{s'}}\left[\xi_{\tilde{\varphi}_s,\varphi_s^*} - 1\right]\right)\right] \qquad \text{if s=s'}$$

$$\xi_{M_s,\tau} = \frac{\sum_{s=1}^{S} \frac{\partial T_s}{\partial \tau} \tau}{wL + \sum_{s=1}^{S} T_s - p^G q_0^G} - \left[\frac{(1-\delta_s)\tau}{(1-\tau)(1-\delta_s\tau)} + (\sigma_s - 1)\left(\xi_{\varphi_s^*,\tau}\left[\xi_{\tilde{\varphi}_s,\varphi_s^*} - 1\right]\right)\right]$$

Using proposition 1, the last term inside the square brackets $(\xi_{\tilde{\varphi}_s,\varphi_s^*} - 1)$ is zero under the Pareto

Figure 2: Elasticities under the Pareto and log-normal distribution with associated pdfs. Pareto distribution parameters selected to match the mode and mean of the log-normal distribution



distribution assumption. The deleted factors capture changes on firms dispersion, which is a measure of the impact on the degree of sector competitiveness.

Building upon the previous results, I provide ordinal statements regarding ξ_M under the two distributional assumptions of productivity.

Proposition 2 *Assume that the government runs a balanced budget* $\left(\sum_{s=1}^{S} T_s = p^G q_0^G\right)$. *Let* ξ^P *be the elasticities from assuming a Pareto distribution and* ξ^{log} *be the elasticities obtained under a log-normal distribution.*

- *Let* $s \neq s'$, *then* $\xi^{log}_{M_s,\delta_{s'}} = \xi^{P}_{M_s,\delta_{s'}} = 0$
- *Let* $s = s'$ *then* $\xi^{log}_{M_s,\delta_{s'}} < \xi^{P}_{M_s,\delta_{s'}}$. *Furthermore if* $\delta > (\leq) 0$ *then* $\xi^{P}_{M_s,\delta_{s'}} > (\leq) 0$

Proof. See Appendix B.2 ■

Thus $\xi^{log}_{M_{s'},\delta_{s'}}$ is always less than its Pareto counterpart, but its sign is not always determined. When $\delta_{s'} \leq 0$, an increase in the depreciation allowance rate results in a decrease in the number of firm, furthermore, the magnitude of change under the log-normal is greater. However, it is not possible to sign $\xi^{log}_{M_{s'},\delta_{\prime}}$ when $\delta_{s'} > 0$. The last case is intriguing since it opens the possibility that the direction of change for $M_{s'}$, following changes to $\delta_{s'}$, will have different signs for each distributional assumption of productivities.

Turning to the statutory corporate tax rate:

Proposition 3 *Assume that the government runs a balanced budget* $\left(\sum_{s=1}^{S} T_s = p^G q_0^G\right)$. *Let* ξ^P *be the elasticities from assuming a Pareto distribution and* ξ^{log} *be the elasticities obtained under a log-normal distribution.*

- *If* $\delta_s \leq 1$ *then* $\xi^{log}_{M_s,\tau} < \xi^{P}_{M_s,\tau} \leq 0$.
- *If* $\delta_s > 1$ *then* $\xi^{log}_{M_s,\tau} < \xi^{P}_{M_s,\tau}$ *Furthermore,* $\xi^{P}_{M_s,\tau}$ *is positive but* $\xi^{log}_{M_s,\tau}$ *can't be signed.*

Proof. See Appendix B.3 ■

Interpretation and consequences of proposition 3 are similar to those of proposition 2 so they are omitted.

3. Optimal Corporate Tax Rates in the Closed Economy

This section describes and solves the optimal corporate tax rate under a fiscal framework designed to capture the essential features of the corporate tax codes observed in the real world.

The government problem is to choose the optimal *effective* corporate tax rates that raise sufficient tax revenue to finance an exogenous given government expenditure while maximizing aggregate welfare. Let $E(\tau, \{\delta_s\}_1^S)$ be the set of optimal consumption and price vectors for a given τ and $\{\delta_s\}_1^S$. The government problem is:

$$\underset{\tau, \{\delta_s\}_1^S}{max} \quad L q_0^G + L \prod_{s=1}^{S} Q_s^{\alpha_s}$$

such that

$$\sum_{s=1}^{S} T_s \geq p^G q_0^G$$

$$(q^*, p^*) \quad \in \quad E(\tau, \{\delta_s\}_1^S)$$

$$0 < \tau \leq 1 \qquad \delta_s < 1/\tau \qquad \forall s \in S$$

Note that the government must raise tax revenue using two instruments: a statutory corporate tax rate and depreciation allowance rates. In one hand, changing τ affects the equilibrium productivity in all sectors and, consequently, the price indexes that affect welfare. On the other hand, it can affect a specific sector by modifying the relevant depreciation allowance rate, thereby enhancing or mitigating the effects of τ in the sector equilibrium productivity and number producing firms.

The FOCs of the government optimization problem in terms of elasticities:

$$\sum_{s=1}^{S} \alpha_s \left(\frac{1}{1-\sigma_s} \xi_{M_s,\delta_{s'}} - \mathcal{I}_{s=s'} \left(\xi_{\tilde{\varphi}_s,\varphi_s^*} \xi_{\varphi_s^*,\delta_{s'}} \right) \right) \leq \delta_{s'} \tilde{\lambda} \sum_{s=1}^{S} \frac{\partial T_s}{\partial \delta_{s'}} \qquad \forall s' \in S \tag{3.1}$$

$$\sum_{s=1}^{S} \alpha_s \left(\frac{1}{1-\sigma_s} \xi_{M_s,\tau} - \xi_{\tilde{\varphi}_s,\varphi_s^*} \xi_{\varphi_s^*,\tau} \right) = \tau \tilde{\lambda} \sum_{s=1}^{S} \frac{\partial Ts}{\partial \tau} \tag{3.2}$$

$$\lambda \left(q_0^G - \sum_{s=1}^{S} T_s \right) = 0 \tag{3.3}$$

$$\frac{\mathbb{P}\lambda + 1}{Y} = \tilde{\lambda} \tag{3.4}$$

where λ is the Lagrange multiplier associated with the government budget constraint, $\mathcal{I}$ is the indicator function, and $\mathbb{P}$ is the economy wide price index.[8] The second equation holds with equality since I assume positive government spending ($q_0^G > 0$) and tax revenue can't be positive unless $\tau > 0$.

The modified FOCs clearly show that the productivity distribution assumption will play a central role in the solutions to the optimal tax problem. As shown in section 2.2, the elasticities appearing in the above equations are significantly different across the two distributional assumptions, particularly $\xi_{\tilde{\varphi}_s,\varphi_s^*}$ which is fixed to unity under Pareto and variable under log-normal.

I proceed to show the optimal tax/depreciation rates for the two different distributional assumptions of productivities for the case with a binding government budget constraint.[9] The Lagrange multiplier associated with a binding government budget constraint is:

Proposition 4 *Assuming that the government budget constraint is binding ($\lambda > 0$), the value of $\tilde{\lambda}$ is:*

$$\tilde{\lambda} = \frac{\sum_{s=1}^{S} \dfrac{\alpha_s}{\sigma_s - 1}}{wL \sum_{s=1}^{S} \dfrac{\alpha_s}{\sigma_s} - p^G q_0^G}$$

Proof. See Appendix B.4 ■

[8]
$$\mathbb{P} = \Pi_{s=1}^{S} \left(\frac{\mathbb{P}_s}{\alpha_s} \right)^{\alpha_s}$$

[9] Appendix A.1 present the derivation of the optimal rates.

3.1. Optimal tax policy under Pareto

Assume productivities follow a Pareto distribution with CDF $Z_s(x) = 1 - \left(\frac{\varphi_{min,s}}{x}\right)^{k_s}$. The optimal statutory corporate tax rate and depreciation allowance rates are:

$$\xi_{\varphi_s^*,\delta_s} = \xi_{\varphi_s^*,\tau} = \frac{-\tau\delta_s}{k_s(1-\delta_s\tau)} \tag{3.5}$$

$$1-\tau = \left[\sum_{s=1}^{S}\frac{\alpha_s}{k_s}\right]\left[\tilde{\lambda}wL\sum_{s=1}^{S}\frac{\alpha_s\rho_s}{k_s}\right]^{-1} \tag{3.6}$$

$$1-\delta_{s'}\tau = \left(\sum_{s=1}^{S}\frac{\alpha_s}{k_s}\Big/\sum_{s=1}^{S}\frac{\alpha_s\rho_s}{k_s}\right)\rho_{s'} \tag{3.7}$$

Proposition 5 *The differences between sector depreciation rates are proportional to the elasticities of substitutions between their sectors. Furthermore, the ratio of usercosts is solely a function of such elasticities:* $\frac{u_{s'}}{u_s} = \frac{\rho_{s'}}{\rho_s}$.

The above proposition says that in an economy with Pareto distributions, firms in sectors with higher elasticities of substitutions get lower depreciation allowance rates relative to sectors with lower elasticities of substitution. Going a step further, the elasticity of substitution within each sector is the *sole* driver for the targeted depreciation allowance rates.

To explain the mechanics behind this result, consider two different sectors s', s with the same shape parameter k but different elasticities of substitution and, without loss of generality, assume that $\sigma_{s'} > \sigma_s$. The key variable that drives the equilibrium results is $h^{\sigma-1}$, which appears in the ZP condition and the formulas for M_s (equation 2.9). By proposition 1 we know that under a Pareto distribution, $h^{\sigma-1}$ is constant regardless of the equilibrium value of φ^*; moreover, this variable is increasing in σ since in equilibrium $h_s^{\sigma_s-1} = \frac{k_s}{k_s-(\sigma_s-1)}$.

First, because h is constant under Pareto, changes in the tax instruments only modify the ZPC equation via the factor $(1-\delta\tau)$. Since this factor is multiplied by $(h^{\sigma-1}-1)$, changes in the tax instruments will have a greater effect in the productivity cutoff in sector s' relative to s. In subsection 2.2 we saw that decreasing δ_s increases the productivity cutoff φ_s^*; therefore, the government gives the smaller depreciation allowance rate to sector s' since it gains more than in sector s in terms of equilibrium productivities. Productivity increases decrease the price index, which translates to higher welfare.

Second, there is a trade-off from having a high elasticity of substitution as it is negatively related to the number of firms in equilibrium. The denominator in equation 2.9 shows that the government could improve the number of firms by decreasing the usercost (u_s). By increasing the depreciation allowance rate, the government reduces the usercost faced by firms. The government deploys this strategy in sector s as it has a higher impact on M relative to sector s'. Hence, the government aims to decrease the price index for sector s by increasing M_s.

Given the importance of the elasticity of substitution in determining the equilibrium outcomes, the next proposition contains a surprising result regarding the relationship between depreciation allowance rates across all sectors.

Proposition 6 *Let the economy consist of S sectors with equal expenditure shares: $\alpha_s = \bar{\alpha} = 1/S$. If productivities are Pareto distributed with a homogeneous shape parameter $\bar{k}$ across sectors, then $\sum_{s=1}^{S} \delta_s^P = 0$.*

Proof. See Appendix B.5 ■

The above result says that regardless of the degree of sector heterogeneity, as long as sector expenditure shares and the Pareto distribution shape parameters are homogeneous, then the depreciation allowance rates will add up to zero. Notice that there is no condition on the distribution parameter φ_{min} only on the shape parameter k since h is only a function of the latter.

3.2. Optimal tax policy under log-normal

Now, assume productivities follow a distribution $Z_i \sim log\ \mathcal{N}(m_s, v_s)$. In this economy, the average productivity in equilibrium is:

$$\begin{aligned}\tilde{\varphi}_s^{\sigma_s - 1} &= \exp\left(m_s(\sigma_s - 1) + \frac{((\sigma_s - 1)v_s)^2}{2}\right)\frac{\Phi((\sigma_s - 1)v_s - d_s)}{\Phi(-d_s)} \\ &= A_s g_s(\varphi_s^*)\end{aligned}$$

where Φ is the standard normal distribution CDF and $d_s = \dfrac{log(\varphi_s^*) - m_s}{v_s}$. The marginal productivity cutoff has to be solved numerically using:

$$\frac{A_s g_s(\varphi_s^*)}{(\varphi_s^*)^{\sigma - 1}} = \frac{\psi F_{e,s}}{(1 - \delta_s \tau)\Phi(-d_s) f_s} + 1$$

Even though the optimal tax rates for this economy don't have closed-form solutions, it is possible to make some analytical comparisons of these optimal tax rates with those obtained under the Pareto distribution.

First, consider the elasticity of the productivity cutoff to deviations in τ, δ:

$$\xi_{\varphi_s^*,\delta_s} = \xi_{\varphi_s^*,\tau} = \frac{\psi F_{e,s}}{X_s(1-\sigma_s)}\left(\frac{\tau\delta_s}{1-\tau\delta_s}\right) \tag{3.8}$$

$$X_s = \psi F_{e,s} + (1-\delta_s\tau)\Phi(-d_s)f_s \tag{3.9}$$

Unlike the case of Pareto distributions, these elasticities are dependent on the fixed cost of production and entry. Therefore, the government's ability to influence the equilibrium productivity is amplified or dampened by sector-specific fixed costs unrelated to the productivity distribution.

The conditions to obtain optimal depreciation allowances equal to zero differ significantly across the two productivity distribution assumptions. The following proposition specifies such conditions:

Proposition 7 *Let $q_0^G > 0$ and $\lambda > 0$. The conditions for $\delta_s = 0 \quad \forall s$ are:*

(i) Pareto distribution: The shape parameter and elasticity of substitution must be equal across sectors ($k_s = \bar{k} \quad \forall s \in S$, $\sigma_s = \sigma \quad \forall s \in S$).

(ii) Log-normal distribution: The sectors in the economy must be symmetric in all respects.

Proof. See Appendix B.6. ■

The condition placed on the Pareto model is significantly weaker from that of the log-normal model. Once again, this is a result that under Pareto h is fully determined by σ, k and of constant value regardless of the equilibrium productivity cutoffs. As mentioned previously, the optimal rates in the Pareto model do not depend on fixed or entry costs; therefore, there is no need to impose symmetry on them. In contrast, the optimal rates in the log-normal environment are affected by such costs; therefore, a stringent condition is needed to force all optimal depreciation allowances to zero.

Proposition 7 highlights one of the most important policy implications of computing their optimal corporate tax rates using the formulas derived under, the literature standard, Pareto distribution assumption versus, the data favored, log-normal distributional. Non-zero depreciation allowance rates imply effective corporate tax rates that diverge from the statutory rate (τ). Therefore, the optimal fiscal instruments derived under the log-normal model indicate a more significant role of government in reallocating resources across sectors to maximize aggregate welfare. The result highlights that the government in the log-normal scenario has an additional transmission channel of their policies, via alterations of h, which allows the government to take full advantage of sector asymmetries by using δ more heavily than under Pareto.

3.3. Optimal Fiscal Instruments as Functions of Select Parameters

I continue by exploring the difference in responses of optimal depreciation and tax rates to changes in the elasticity of substitution, country size, government spending, and fixed costs. To make the exposition clearer, I conduct the numerical exercises in a version of the model with only two almost identical sectors whose only difference lie in their elasticity of substitution σ_s. Table 1 reports the parameter values, which are standard except for the productivity parameters. The log-normal distribution parameters (m_s, v_s) are set to average values of the empirical estimates of Section 6.2, while the Pareto distribution parameters $(k_s, \varphi_{min,s})$ are set to match the mean and variance of the log-normal distribution.[10]

The take away from all these response functions is twofold. First, the productivity distribution assumption is not essential when sectors are identical but becomes critical when the economy is composed of asymmetric sectors. Moreover, the divergence between the optimal rates implied by each distributional assumption increases with the degree of heterogeneity between sectors, especially when the asymmetry involves the elasticity of substitution. Second, if a sector experiences changes in fixed or entry costs, then each distributional assumption will result in entirely different responses for the depreciation allowance rates and consequently, the corporate tax rate.

Although a full symmetric case is not used as a baseline, the response functions in Figure 3 contain a point ($\sigma_2 = 2.5$) for which both sectors are completely symmetric. As stated in proposition 7, this special case generates depreciation rates equal to zero for both sectors regardless of distributional assumption. Intuitively, when both sectors are completely symmetrical, they can be aggregated into a single sector with the same properties. In this case, the government cannot improve upon the free market ("first best") outcome by shifting resources across the sectors. The free market equilibrium productivity is that of Melitz (2003), which my model attains when setting δ or τ to zero. Since $q_0^G > 0$, the statutory corporate tax rate (τ) is strictly positive therefore depreciation rates are optimally zero.

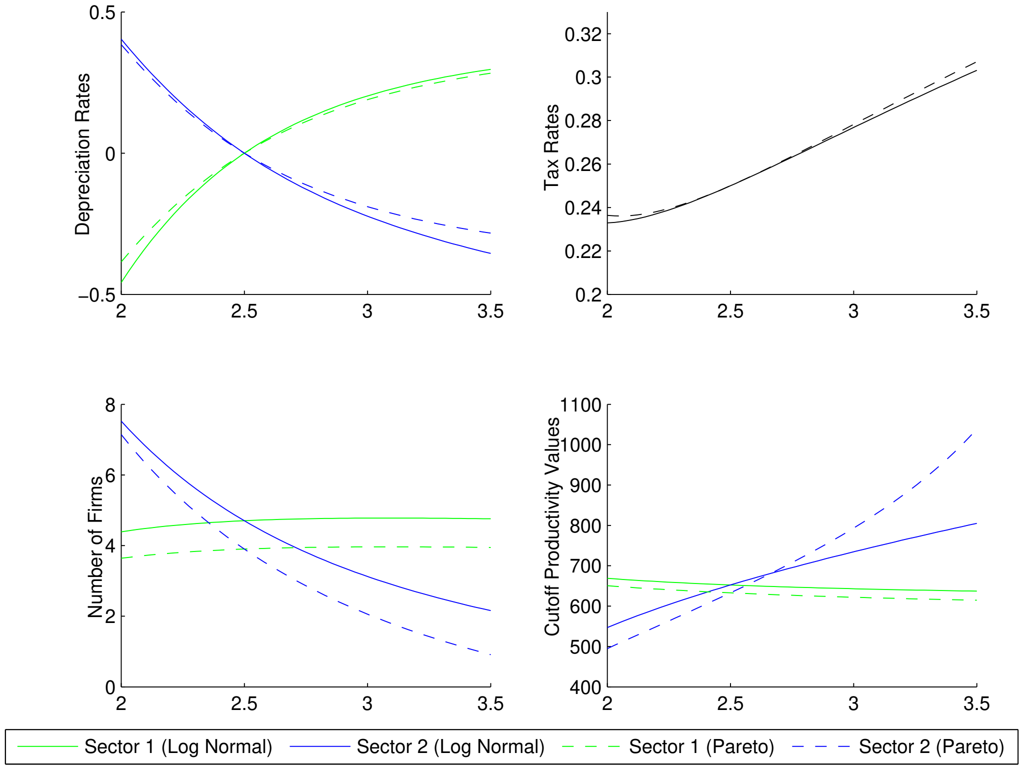
I now describe the sensitivity of optimal tax instruments rates and equilibrium responses as the elasticity of substitution in sector 2 varies in the interval $[2, 3.5]$, while sector 1 remains fixed at $\sigma_1 = 2.5$. Optimal depreciation rates produced under log-normal productivities exhibit a larger degree of responsiveness to

[10] By matching the variances, we implicitly impose a finite variance for the Pareto distribution, which implies that k is strictly greater than 2. Solving for the Pareto distribution parameters leads to a quadratic polynomial for k; choosing the non-negative root gives the following formulas:

$$k_s = 1 + \sqrt{\frac{\exp(v_s^2)}{\exp(v_s^2) - 1}}$$

$$\varphi_{min,s} = \exp(m_i + \frac{v^2}{2})\frac{k_s}{k_s - 1}$$

Figure 3: Effects of Changes in the Elasticity of Substitution for sector 2



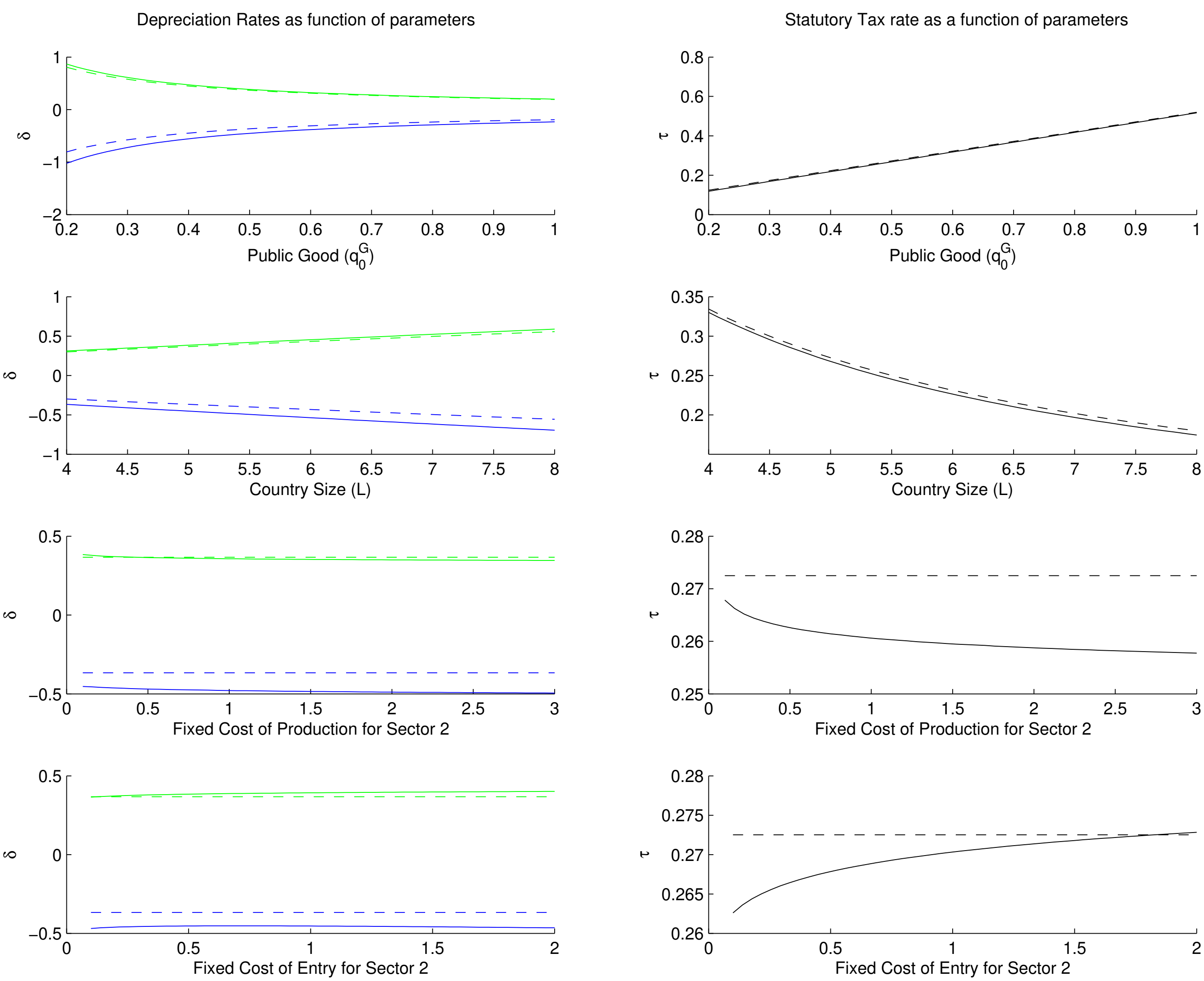
changes in σ_2 when compared to their Pareto counterparts; the divergence between such rates increases with the distance between σ_1 and σ_2. This divergence occurs even though the Pareto and log-normal productivity distributions have the same unconditional mean and variance. Thus, the divergence is mainly a result of the extra channel of effect (through $\xi_{\tilde{\varphi},\varphi^*}$) that the log-normal setting posses.

In contrast to the optimal depreciation allowance rates, the response functions for τ are more responsive when Pareto distributions are assumed. The take away of this analysis is that a policymaker in an environment with Pareto distributed productivity will optimally distribute the burden of taxation more evenly across the sectors than the log-normal case. Importantly, the relatively small differences in observed tax and depreciation allowance rates have significant implications for the number of firms in each sector and the efficiency of the marginal firm.

Across both productivity distributions, the sector with the smallest elasticity of substitution receives the lesser of the depreciation allowances. In proposition 5, I explained the mechanics for this property for the Pareto case. The same applies to the log-normal environment with the addition that the term $h^{\sigma-1}$ is variable for this setting; hence, depreciation allowance rates exhibit a more drastic change.

Next, figure 4 shows the response functions for changes in government spending, country size, entry cost, and fixed costs of production. As government expenditure increases, the budget constraint becomes tighter,

Figure 4: Depreciation and tax rates as functions of different variables



which limits the ability of governments to exploit the variability of productivity distributions; hence, we observe convergence in the optimal values of δ and τ under the two distributional assumptions. When L increases, the corporate tax rate decreases since firms in both sectors earn higher revenues. The last two rows of the figure 4 show the responses to changes in the fixed cost of production and entry in sector 2. Under the Pareto model, optimal δs are invariant to changes in fixed costs, In contrast, optimal δs and τ under the log-normal model respond to changes in fixed costs.

3.4. Inefficient outcomes from assuming a Pareto distribution

To finalize this section, I study the welfare implications of a government misspecifying the productivity distribution when computing their optimal depreciation and corporate tax rates. Based on recent theoretical and empirical research, as well as the empirical evidence presented in section 6.2, I posit that countries contain firms that draw their productivities from a log-normal distribution and conduct the following experiment. First, I compute the optimal δs and τ using the formulas implied by the model using Pareto distributions. I call these the "null" optimal rates and use them used to compute the equilibrium for the economy, even though firms draw their productivities from a log-normal distribution.[11] Next, I repeat the process but using the "alternative" formulas for the optimal rates, i.e., the formulas under the log-normal assumption. I then compare the outcomes of the two models. Welfare under both models is comparable since the amount of public good (q_0^G) constant, and any difference between government expenditures and tax revenues is transferred or taken from households through a lump-sum tax. Experiments are conducted under five different scenarios, with results reported in Table 1. The top lines report the outcome using the "null" optimal tax rates while outcomes using the "alternative" tax rates are directly underneath.[12]

The almost symmetric scenario shows that using the more straightforward Pareto formulas for the optimal δs and τ carries a 0.14% loss in welfare relative to using the "alternative" formulas. The "alternative" and "null" models have almost identical equilibrium outcomes, except for the depreciation allowances which are non-symmetric across sectors for the log-normal case.

The next two scenarios contain sector asymmetries in the fixed cost of production and entry. For these scenarios, the penalties in welfare are more substantial than in the almost symmetric case; albeit, the equilibrium variables for both models are similar. The optimal δ, τ under Pareto are the same as those of the almost symmetric scenario but, in the log-normal case, the corporate tax rates change across scenarios (proposition 7). The adaptation of the fiscal instruments to changes in fixed cost drives the improvement in welfare. Thus, while the formulas for computing the optimal fiscal instruments under the log-normal model are more complicated, they provide non-trivial welfare improvements relative to using the formulas derived under the Pareto assumption.

The next scenario increases the difference between the sectors' elasticity of substitution. This scenario generates the most significant losses in welfare from using the "null" rates in an economy whose firms draw their productivity from a log-normal distribution. The welfare loss is over 2%, which is significantly higher than any of the losses in the previous scenarios. Moreover, the equilibrium outcomes and optimal fiscal

[11] These rates are not the solution to the government problem and therefore the budget constraint may not hold with equality, i.e $\Sigma T_s \neq p^G q_0^G$. Hence, the number of firms is the solution to the system of equations:

$$M_{s'} = \frac{\alpha_{s'}(wL + \Sigma_{s=1}^{S} T_s - p^G q_0^G)}{\bar{r}_{s'}} \qquad s' = 1, 2$$

[12] We continue to set the Pareto distribution parameters by matching the unconditional mean and variance to that of the log-normal distribution.

instruments values differ significantly between the two models. The policies obtained from a log-normal rely on targeting specific sectors at different rates instead of heavily readjusting τ, as is the case with the Pareto assumption. These results, coupled with empirical estimates showing a high variability of σ across sectors (e.g., Broda and Weinstein (2006), Feenstra et al. (2018)), illustrates the importance of computing the optimal depreciation and tax rates using the proper distributional assumption.

In conclusion, the analytically convenient assumption that productivities follow a Pareto distribution is not innocuous in the context of corporate tax policy.

4. Open Economy

This section extends the model into the open economy to study the linkage between corporate taxes and export status, and provide a basis to explain conflicting empirical results regarding this linkage. In my model, modifications to the statutory corporate tax rate alone generate an ambiguous change in the probability of becoming an exporter. The value of the depreciation allowance rate determines the direction of change.

Additionally, including corporate taxes can solve a critical issue of the multi-sector Melitz model regarding unilateral liberalization of some sectors.[13] The evidence suggests that even a unilateral liberalization results in increased productivity for the sectors whose trade barriers decreased.[14] In theory, Demidova and Rodríguez-Clare (2013) find that a one sector Melitz model generates such implication; however, Segerstrom and Sugita (2015) find that such implication does not hold in the multi-sector version. They find that such model generates the reverse implication under very general conditions. My model can reconcile the theory and empirical evidence by accounting for changes in effective corporate tax rates faced by specific sectors, which offsets/enhance the productivity gains from a unilateral tariff reduction.

The next paragraphs contain only the key elements and results of the model with two countries with identical utilities. Appendix C contains a general model with N countries and asymmetric utility parameters (α, σ).

4.1. Setup, Aggregation and Equilibrium

Household preferences in both nations have the same functional form described in section 2, and I assume no labor migration. Since consumers can now buy products from other countries, I use x_{jis} to represent a variable from country j with final market in country i, for sector s.

[13] Unilateral liberalization refers to a single country reducing their trade barriers/cost to imports

[14] See for example Trefler (2004)

The timing of decisions by the firm is the same as in the closed economy, but firms serving the domestic market can choose to serve the foreign country via exports. Shipping goods across countries involves an iceberg trade cost $\theta_{jis} \geq 1$; and exporting firms pay a fixed investment cost (f_{jis}) every period. This additional fixed cost is also subject to the depreciation allowance rate (δ_{js}). The after-tax profit formula for a firm in country j is:

$$\pi_{js}(\varphi) = (1-\tau_j)\left(\frac{r_{jjs}(\varphi)}{\sigma_s} - u_{js}w_j f_{jj} + \mathcal{I}_{export}\left(\frac{r_{jis}(\varphi)}{\sigma_s} - u_{js}w_j f_{jis}\right)\right) \tag{4.1}$$

$$r_{jis}(\varphi) = \left(\frac{p_{jis}(\varphi)}{\mathbb{P}_{is}}\right)^{(1-\sigma_s)} Y_{is} \tag{4.2}$$

Define φ^*_{jj} as the cutoff productivity levels for the marginal firm that decides to serve the domestic market and φ^*_{ji} as the productivity level of the marginal firm that chooses to export to country i. Using $\tilde{\varphi}(\ )$ (equation 2.7) define the average productivity of all firms producing in j ($\tilde{\varphi}_{jj}$) and the average productivity of firms that export their goods to i ($\tilde{\varphi}_{ji}$):

$$\tilde{\varphi}_{jj} = \tilde{\varphi}^j(\varphi^*_{jj}) \qquad\qquad \tilde{\varphi}_{ji} = \tilde{\varphi}^j(\varphi^*_{ji})$$

The number of producing firms in sector s, based in country j, is M_{js} with a subset $M_{jis} = \kappa^x_{jis} M_{js}$ serving country i via exports; where κ^{ex}_{ji} is the conditional probability of becoming an exporter.[15] Hence, the total amount of products available to consumers in country j is $M^j_{tot,s} = M_{js} + M_{ijs}$.

With the above, the price index and the average productivity of firms *selling* in country j is:

$$\tilde{\varphi}^j_{tot,s} = \left[\frac{1}{M^j_{tot,s}}\left(M_{js}\left(\tilde{\varphi}_{jj}\right)^{\sigma_s-1} + M_{ijs}\left(\hat{\theta}^{-1}_{ijs}\tilde{\varphi}_{ijs}\right)^{\sigma_s-1}\right)\right]^{\frac{1}{\sigma_s-1}} \tag{4.3}$$

$$\mathbb{P}_{js} = \left(M^j_{tot,s}\right)^{\frac{1}{1-\sigma_s}} p_{jjs}(\tilde{\varphi}^j_{tot,s}) \tag{4.4}$$

where $\hat{\theta}_{ijs} = \dfrac{w_i\theta_{ijs}}{w_j}$. The total average productivity ($\tilde{\varphi}_{tot,s}$) is the weighted average of mean productivities of all domestic firms and foreign firms selling products in country j.

The sector price index formulas are needed to solve for the equilibrium since the new zero profit condition contains domestic and export productivity cutoffs linked through the sector price index. To be more clear,

[15] $\kappa^x_{jis} = \dfrac{1 - Z_{js}(\varphi^*_{jis})}{1 - Z_{js}(\varphi^*_{jjs})}$

the new ZP condition is:

$$\bar{\pi}_{js} = (1 - \delta_{js}\tau_j)\left[w_j f_{jjs}\left(\left(\frac{\tilde{\varphi}_{jjs}}{\varphi^*_{jjs}}\right)^{\sigma_s - 1} - 1\right) + \kappa^{ex}_{jis} w_{js} f_{jis}\left(\left(\frac{\tilde{\varphi}_{jis}}{\varphi^*_{jis}}\right)^{\sigma_s - 1} - 1\right)\right] \tag{4.5}$$

and to solve φ^*_{jis} it must be expressed as a function of φ^*_{jjs}:

$$\varphi^*_{jis} = \left[\frac{M^i_{tot,s}}{M^j_{tot,s}}\right]^{\frac{1}{\sigma_s - 1}} \frac{\tilde{\varphi}^i_{tot,s}}{\tilde{\varphi}^j_{tot,s}} \left[\frac{Y_{js}}{Y_{is}} \frac{f_{jis}}{f_{jjs}}\right]^{\frac{1}{\sigma_s - 1}} \hat{\theta}_{jis}\varphi^*_{jjs} \tag{4.6}$$

Notice that the above equation expresses the export productivity cutoff for country j as a function of other productivity cutoffs, including those of country i. Many papers at this point invoke a symmetry assumption across the countries making the above sufficient to pin down the equilibrium productivities. However, in my model, even if countries were completely symmetric in all their parameters but one of their corporate tax rates, it would generate different domestic cutoffs which translate into different equilibrium outcomes between the countries. Borrowing from Segerstrom and Sugita (2015), I use the relationship between domestic and import productivity cutoffs:

$$\varphi^*_{jis} = \left(\frac{u_{js} w_j f_{jis}}{u_{is} w_i f_{ii}}\right)^{\frac{1}{\sigma_s - 1}} \hat{\theta}_{ji}\varphi^*_{ii} \tag{4.7}$$

to convert equation 4.6 into a function of φ^*_{jj} only.

Lastly, the solution to the number of firms completes the description of the equilibrium. Labor used for production remains $r(\varphi) - \pi(\varphi) - t(\varphi)$ and we can use the same procedure as in section 3 to obtain aggregate revenue $R = wL + \sum T - p^G q^G_0$. Therefore, the equilibrium is the solution to a $S \times 2 \times 2$ system of simultaneous equations consisting of the following 2 equations for each sector and each country:

$$ZP_{s'} = FE_{s'} \tag{4.8}$$

$$M_{js'} = \frac{\alpha_{js'}(w_j L_j + \Sigma^S_{s=1} T_{js} - p^G_j q^G_{j0})}{\sigma_{js'} u_{js'} w_j \left(f_{jjs'} h^{\sigma_{s'} - 1}_{jjs'} + \kappa^{ex}_{jis'} f_{jis'} h^{\sigma_{s'} - 1}_{jis'}\right)} \tag{4.9}$$

where $h_{jj} = \tilde{\varphi}_{jj}/\varphi^*_{jj}, \quad h_{ji} = \tilde{\varphi}_{ji}/\varphi^*_{ji}$

4.2. Corporate Tax Rates and Export Status

This subsection provides a detailed account of the relationship between the export productivity cutoffs and corporate tax rates. I find that the conditional probability of exporting is negatively related to the depreciation rate (in the source country), but the relationship with the statutory corporate tax rate is ambiguous. The first part of the result is not surprising as increasing δ decreases the cost of f_{ji} which incentives firms to enter the export market. However, the direction of change for modification in τ is ambiguous as it depends on the level of δ.

The effects of changing δ, τ on the probability of exporting (κ^{ex}) are given by:

$$\frac{\partial \kappa^{ex}_{jis}}{\partial y} y = \kappa^{ex}_{jis} \left(\Upsilon(\varphi^*_{jjs}) \xi_{\varphi^*_{jjs}, y} - \Upsilon(\varphi^*_{jis}) \xi_{\varphi^*_{jis}, y} \right) \qquad \text{for y} = \tau, \delta_s \tag{4.10}$$

$$\Upsilon_{js}(x) = \frac{z_{js}(x)}{1 - Z_{js}(x)} x \tag{4.11}$$

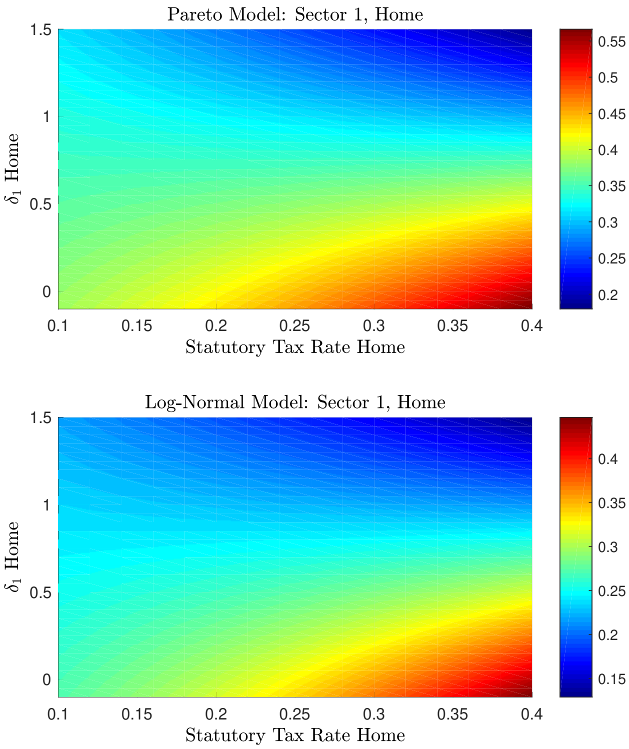
where $\Upsilon(x)$ has the following properties:

- If $Z_{js} \sim Pareto(k_{js}, \varphi_{min})$ then $\Upsilon(\varphi) = k_{js}$ for any φ in the support of Z_{js}.
- If $Z_{js} \sim log\mathcal{N}(m_{js}, v_{js})$ then $\Upsilon(\varphi)$ is an increasing function.

The above shows, once again, that distributional assumptions about productivity are important for the comparative statics of the model. A constant versus increasing Υ has implications for the effects of corporate tax changes on the probability of becoming an exporter. The next section shows that, for the special case of symmetric countries, changes in the fiscal instruments do not affect the probability of exporting (κ) for the model with Pareto distributions; this invariability property is not present when assuming log-normal distributions. For the general case (asymmetric countries), the effects on κ, following changes to tax rates, are determined by the difference between the domestic and export productivity cutoff elasticities. However, the subtraction's terms will be equally weighted for the Pareto case but, under the log-normal assumption, the export cutoff elasticity has a higher weight.

Figure 5 illustrates the relationship between corporate tax rates and the probability of export. The panel presents heat maps for κ_{ji1}: the probability of export for firms in sector 1, country j; as a function of τ_j and δ_{j1}. The export probabilities come from solving the equilibrium for two countries (Home and Foreign) whose parameters are equal to those of the almost symmetric scenario. A surface plot of κ_{ji1} is generated by evaluating the model at grid points spawn by τ_j, δ_{j1}. The left graphs in the panel show that increasing the depreciation allowance rate (δ_{1j}) results in a decrease in the propensity to export by firms in country j, but the relationship between the statutory tax rate (τ_j) and the probability of export is ambiguous. In the graphs, we observe that increasing τ_j increases the probability of exporting, but only when the value of δ_{j1} is below a certain threshold. In contrast, if δ_{j1} is above such threshold, the probability of export decreases with the statutory corporate tax rate.

Figure 5: Probability of exporting, for firms in both countries under the Pareto and log-normal assumption, as the Home's depreciation allowance rate and statutory tax rate varies.



The reason behind the ambiguous effect goes back to the movement of the ZP condition in the closed economy, which was positive for $\delta > 0$ but negative for $\delta < 0$. In the open economy the new ZP condition also contains the term φ^*_{ji} which is determined by the ratio of usercosts across countries; thereby, the critical value for δ at which the relation between τ and productivity cutoffs flips is different from zero.

The relation portrayed in Figure 5 bridges two conflicting empirical findings regarding corporate tax effects on export dynamics. First, Bernini and Treibich (2016) compare the export dynamics of French small-medium firms, whose statutory tax rater fell from 33.33% to 15% for 2001 to 2003, to those of large firms, whose tax rate did not change. They find a negative correlation between corporate tax rates and the probability that firms engage in export activities.[16] As we have seen in Figure 5, my model generates such relationship but only when the depreciation allowance rate is above a threshold. On the other hand, Federici and Parisi (2014) use data from Italian firms, for the years 2004 to 2006, to show that export propensity is positively associated with corporate taxation, which in their study is a measure of firms' specific effective tax rate. In my model, this would translate to a negative relationship between the sector depreciation allowance rate and the probability of exporting, which is what we observe in Figure 5.[17]

[16] Alessandria and Choi (2014) also finds a negative relationship between corporate taxation and export *growth*. Liu and Lu (2015) find that reductions in value-added taxes in China resulted in increased exports.

[17] Increasing δ_s allows firms in sector "s" to increase their reduction in taxable income and thereby reduce their tax liability.

Adding corporate taxation to a multi-sector Melitz model ameliorates the critique of Segerstrom and Sugita (2015) who find that such model is inconsistent with the data. In the data, sector productivity increases more strongly in liberalized sectors than in non-liberalized sectors; however, the multi-sector Melitz model generates the inverse relationship under fairly general conditions. Using equation 4.7, we can observe that the effects of a unilateral decrease in trade costs (θ) can be offset through corporate tax changes in either country. Hence, the critique of Segerstrom and Sugita (2015) regarding the implication of a multi-sector Melitz model can be attenuated with the inclusion of the corporate tax framework presented in this paper.

While the question of interest was on the relationship between exports and the corporate tax rates, I also show that the model is consistent with the literature results concerning liberalization. Using equation 4.6, we see that liberalization (lower θ) reduces the productivity cutoff to serve country i via exports. The same equation also provides a relationship between market competition and the export productivity required to "carve" space in such market. For example, if many firms are operating in country i or the productivity of such firms is high ($\tilde{\varphi}_{tot,i}$), then the required export productivity cutoff will be higher relative to other less competitive markets.

5. Optimal Corporate Tax Rates in the Open Economy

This section will provide the characterization of the optimal corporate tax rates in the open economy, for a general case; and its solutions, for the particular case of symmetric countries.

Without loss of generality assume $j \neq i$. The optimization problem and associated conditions for country j:

$$\max_{\tau_j, \{\delta_{js}\}_1^S} \quad L^j q_j^G + L_j \prod_{s=1}^{S} Q_{js}^{\alpha_{js}}$$

such that

$$\sum_{s=1}^{S} T_{js} \geq p_j^G q_{j0}^G$$

$$(q^*, p^*) \quad \in \ E(\tau_j, \{\delta_{js}\}_1^S)$$

$$0 < \tau_j \leq 1 \qquad \delta_{js} < 1/\tau_j \qquad \forall s \in S$$

Thus, all else equal, the ratio of taxes paid to profits will decrease (effective tax rate).

Analysis is restricted for the case of a binding constraints leading to the following FOCs:

$$\left(\frac{\alpha_{js}a_{js}^{-1}}{\sigma_{js}-1}\right)\left(\frac{\xi_{M_{js},\delta_{js}}}{\tilde{\varphi}_{jjs}^{1-\sigma_s}}+\frac{\partial\tilde{\varphi}_{jjs}^{\sigma_s-1}}{\partial\delta_{js}}\delta_{js}+\frac{M_{is}}{M_{js}}\hat{\theta}_{ijs}^{1-\sigma_s}\left(\frac{\partial\kappa_{ijs}^{x}}{\partial\delta_{js}}\delta_{js}\tilde{\varphi}_{ijs}^{\sigma_s-1}+\kappa_{ijs}^{x}\left(\frac{\xi_{M_{is},\delta_{js}}}{\tilde{\varphi}_{ijs}^{1-\sigma_s}}+\frac{\partial\tilde{\varphi}_{ijs}^{\sigma_s-1}}{\partial\delta_{js}}\delta_{js}\right)\right)\right)$$
$$=-\tilde{\lambda}^{j}M_{js}\left(\xi_{M_{js},\delta_{js}}\bar{t}_{js}+\frac{\partial\bar{t}_{js}}{\partial\delta_{js}}\delta_{js}\right)\qquad\forall s\in S \tag{5.1}$$

$$\sum_{s=1}^{S}\left(\frac{\alpha_{js}a_{js}^{-1}}{\sigma_{js}-1}\right)\left(\frac{\xi_{M_{js},\tau_j}}{\tilde{\varphi}_{jjs}^{1-\sigma_s}}+\frac{\partial\tilde{\varphi}_{jjs}^{\sigma_s-1}}{\partial\tau_j}\tau_j+\frac{M_{is}}{M_{js}}\hat{\theta}_{ijs}^{1-\sigma_s}\left(\frac{\partial\kappa_{ijs}^{x}}{\partial\tau_j}\tau_j\tilde{\varphi}_{ijs}^{\sigma_s-1}+\kappa_{ijs}^{x}\left(\frac{\xi_{M_{is},\tau_j}}{\tilde{\varphi}_{ijs}^{1-\sigma_s}}+\frac{\partial\tilde{\varphi}_{ijs}^{\sigma_s-1}}{\partial\tau_j}\tau_j\right)\right)\right)$$
$$=-\tilde{\lambda}^{j}\sum_{s=1}^{S}M_{js}\left(\xi_{M_{js},\tau_j}\bar{t}_{js}+\frac{\partial\bar{t}_{js}}{\partial\tau_j}\tau_j\right) \tag{5.2}$$

where a_{js} is a weighted mean of the average productivities of firms selling in country j, sector s, and $(\bar{t}_{js})$ is average tax revenue:

$$a_{js}=\tilde{\varphi}_{jjs}^{\sigma_s-1}+\kappa_{ijs}^{x}\frac{M_{is}}{M_{js}}\left(\hat{\theta}_{ijs}^{-1}\tilde{\varphi}_{ijs}\right)^{\sigma_s-1}$$
$$\bar{t}_{js}=\tau_j\left(w_jf_{jj}(u_{js}h_{jjs}^{\sigma_s-1}-\delta_{js})+w_jf_{ji}\kappa_{ji}^{x}(u_{js}h_{jis}^{\sigma_s-1}-\delta_{js})\right)$$

The FOCs tell us that the government faces a similar problem as in the closed economy section: the left-hand side is the benefit/cost to the average productivity of firms and the right-hand side is the benefit/cost to tax revenue. However, the left-hand side now includes a term for the productivity of importers, which is affected by tax policy in j as stated in equations 4.6 and 4.7. The right hand also includes an additional facto, revenue from exporting products into i, which can be influenced by the fiscal instruments.

The expressions for the elasticity of the number of firms aid in the understanding of the effects of assuming Pareto distributions:

$$\xi_{M_{js},\delta_{js}}=-\left[\frac{-\tau_j\delta_{js}}{1-\tau_j\delta_{js}}+\frac{f_{jjs}\frac{\partial h_{jjs}^{\sigma_s-1}}{\partial\delta_{js}}\delta_{js}+f_{jis}\left(\frac{\partial h_{jis}^{\sigma_s-1}}{\partial\delta_{js}}\delta_{js}\kappa_{jis}^{x}+\frac{\partial\kappa_{jis}^{x}}{\partial\delta_{js}}h_{jis}^{\sigma_s-1}\delta_{js}\right)}{f_{jjs}h_{jjs}^{\sigma_s-1}+\kappa_{jis}^{x}f_{jis}h_{jis}^{\sigma_s-1}}\right] \tag{5.3}$$

$$\xi_{M_{js},\tau_j}=-\left[\frac{(1-\delta_{js})\tau_j}{(1-\tau_j\delta_{js})(1-\tau_j)}+\frac{f_{jjs}\frac{\partial h_{jjs}^{\sigma_s-1}}{\partial\tau_j}\tau_j+f_{jis}\left(\frac{\partial h_{jis}^{\sigma_s-1}}{\partial\tau_j}\tau_j\kappa_{jis}^{x}+\frac{\partial\kappa_{jis}^{x}}{\partial\tau_j}h_{jis}^{\sigma_s-1}\tau_j\right)}{f_{jjs}h_{jjs}^{\sigma_s-1}+\kappa_{jis}^{x}f_{jis}h_{jis}^{\sigma_s-1}}\right] \tag{5.4}$$

Just like in the closed economy, the response of the equilibrium number of firms to changes in τ,δ depend upon the distributional assumptions made. The dependence is clear from the terms $\partial h^{\sigma-1}/\partial x$, which are identical to zero when productivities follow a Pareto distribution. For the case of log-normal distributions, the above elasticities contain an additional term that captures the changes in the export market. These alterations are a combination of effects on the productive term or the "intensive" margin; and the change in the ex-ante probability of entering the export market, the "extensive" margin.

5.1. Symmetric countries

The main result of this subsection shows that under the Pareto distribution assumption, optimal tax rates for the open economy with symmetric countries are identical to those obtained in the closed economy. This odd result is unique to the Pareto model as it generates ex-ante probabilities of exporting that are invariant to changes in corporate tax rates. In contrast, the optimal tax rates under the log-normal distribution assumption are different since governments' power to affect M, φ^* via tax policy diminishes when the country opens to trade.

In this setting, I impose the additional restriction that both countries are entirely symmetric, and both governments set their optimal fiscal policies together. In this case, we can think of countries having a "harmonization" scheme with respect to their statutory corporate tax rates and depreciation allowance rates.[18] To avoid the nuisances of first-player advantages or incentives to deviate from the commonly agreed tax rates, I assume that there is a global planner that sets the tax rates.

The full symmetry assumption allows for a straightforward relationship between the export cutoff and the domestic productivity cutoff.

$$\varphi^*_{jis} = \left(\frac{f_{jis}}{f_{jjs}}\right)^{\frac{1}{\sigma_s - 1}} \theta_{jis}\varphi^*_{jjs} \tag{5.5}$$

$$M^j_{tot,s} = M_{js}\left(1 + \kappa^{ex}_{jis}\right) \tag{5.6}$$

The particular relation of φ^*_{ji} with the domestic productivity cutoff has powerful implications for the optimal tax rates; in particular for the case of Pareto as highlighted in the following lemma:

Lemma 8 *Let $x_s = \tau, \delta_s$, under the symmetric assumption the following holds:*

$$\frac{\partial \kappa^{ex}_{jis}}{\partial x} x = \kappa^{ex}_{jis}\xi_{\varphi^*_{jj},x}\left(\Upsilon_{js}(\varphi^*_{jjs}) - \Upsilon_{js}(\varphi^*_{jis})\right) \qquad x = \tau, \delta_s \tag{5.7}$$

Furthermore,

[18] Ever since the report of the Ruding Committee (Devereux, 1992), the European Commission has been exploring and proposing different schemes to "harmonize" corporate taxation across its members. Bettendorf et al. (2010) explores the economic consequences of such tax reform. Theoretical explorations, such as Conconi et al. (2008) and Haufler and Lülfesmann (2015), find that partial harmonization dominates the outcomes from no harmonization or full harmonization.

- *If $Z \sim$ Pareto then* $\dfrac{\partial \kappa^{ex}_{jis}}{\partial x} x = 0.$

- *If $Z \sim log\mathcal{N}$ then* $\dfrac{\partial \kappa^{ex}_{jis}}{\partial x} x > (<)\ 0$ *if* $\xi_{\varphi^*_{jjs}, x_s} < 0\ (> 0)$. *This derivative is only equal to zero when* $\xi_{\varphi^*_{jjs}, x_s} = 0$ *or as* $\varphi^*_{jj} \to \infty$

Lemma 8 states that, under the Pareto distribution assumption, corporate tax rates are neutral in terms of trade effects. Therefore, any change to the statutory tax rate or the depreciation allowance rate has no impact on the *ex-ante* probability of entering the export market and, consequently, on the export status of firms. The neutral property of corporate tax rates regarding exports goes against empirical evidence.

In contrast, when log-normal productivities are assumed the modifications to tax rates have an effect on the export probabilities and hence on the number of exporters in equilibrium. The intuition for the direction of change is simple. First, assume that τ, δ have a negative effect on the domestic productivity cutoff. Since φ^*_{jis} is a fixed multiple of the domestic cutoff, the probability of obtaining productivity above φ^*_{jis} – conditional on successful entry to domestic market – increases since the right tail of the log-normal distribution is monotonically decreasing. A more intuitive explanation: under the symmetry assumption, the foreign market has become less competitive due to the reduction in average productivities and making it easier for domestic firms to serve the international market via exports.

The invariability of the number of exporters to modifications in the tax rate, under the Pareto assumption, has the following implication:

Proposition 9 *Assume productivities are Pareto distributed. The optimal tax rates for the open economy under the symmetry assumption are exactly equal to those obtained in the closed economy.*

Proof. See Appendix D.1 ■

While proposition 9 states that the optimal formula for τ, δ have not changed in this setting, it doesn't imply that equilibrium outcomes haven't changed. The model still generates gains from trade spawn from the increased productivity of the firms following the opening to trade that enhances competition.

Nonetheless, the implication that optimal taxes remain the same in the opening economy is striking and might be judged as an undesirable property generated by the Pareto distribution. The explanation behind this odd outcome is quite simple. In the closed economy, the Pareto distribution mutes a channel of transmission by precluding the rearrangement of the sector via h, which in this open economy setting extends to the export market via h_{ji}. Hence, the closed and open economy optimal rates are the same since the Pareto distribution assumption eliminates the additional export transmission channels that arise in the open economy setting.

In contrast, export market channels play a significant role in the determination of the optimal tax rates in the model with log-normal distributions. The transition from autarky to trade cuts the power of the government to influence equilibrium outcomes:

Proposition 10 *Let $\xi^C_{\varphi^*_{jjs},x_{js}}$, $\xi^O_{\varphi^*_{jjs},x_{js}}$ be the elasticity of the domestic cutoff productivity in the closed and open economy respectively. If firms draw productivity from a log-normal distribution then the following holds:*

$$|\xi^O_{\varphi^*_{jjs},x_{js}}| < |\xi^C_{\varphi^*_{jjs},x_{js}}| \qquad \forall s \in S \ \text{ and } \ x_{js} = \tau_j, \delta_{js}$$

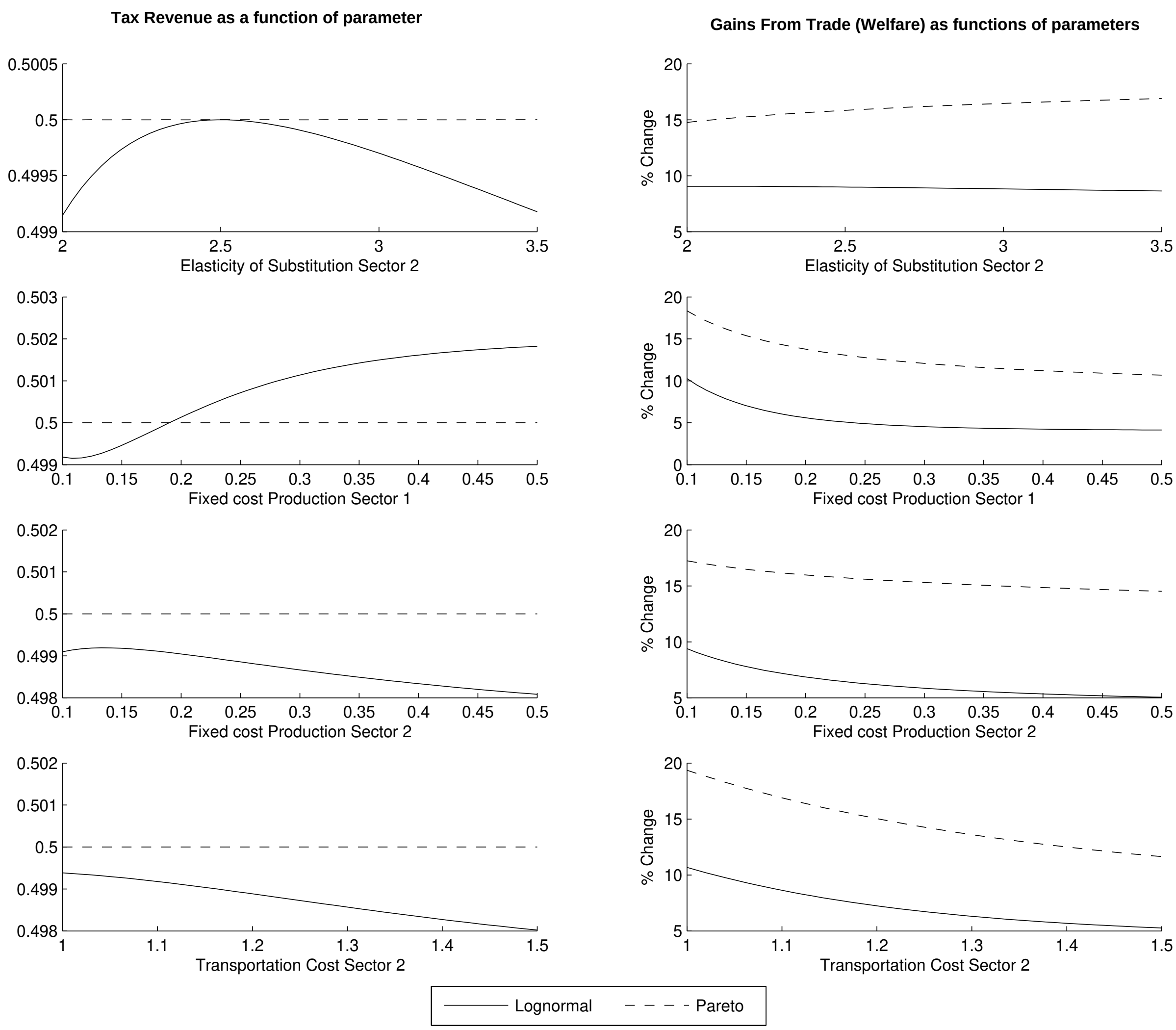
Proof. See Appendix D.2 ■

The discussion of proposition 5 explained how in choosing the optimal fiscal instruments rates, governments make a trade-off between raising productivity in some sectors while increasing the number of firms in others. By proposition 10, under the log-normal assumption, the open economy model diminishes the degree by which governments can influence the equilibrium productivities relative to the closed economy setting. In one hand, this is a "bad" property for sectors with high elasticity of substitution (σ) as the government loses power to raise equilibrium productivity. On the other hand, sectors in which government policies were reducing equilibrium productivity are affected to a lesser degree.

Table 2 contains the results of an economy that opens to trade and retains the optimal tax instrument rates of the closed economy, which is the policy derived in the model with Pareto distributions. Consistent with Head et al. (2014), I find that gains from trade (GFT) under Pareto are significantly higher than those obtained by assuming log-normal distribution of productivities. Moreover, the tax revenue in the log-normal environment decreases for all scenarios, which forces the government to tax households to meet their expenditure. This reduction in disposable income reduces the number of firms, thus diminishing the gains from trade.

To further illustrate the effects on tax revenues from moving into the open economy without changing the corporate tax rates, I present its response function in terms of several parameters in Figure 6. In these graphs, the dash lines correspond to the Pareto distribution assumption while the solid lines are for the economy with a log-normal distribution of productivities. In the first panel, we see that the wedge between public spending and tax revenue increases with the degree of asymmetry in the elasticity of substitution across sectors. Just as in the closed economy, when the sectors are completely symmetric, there is no difference in the optimal tax rates between the Pareto and log-normal distribution assumptions. In term of the fixed cost of production, the tax revenue increases with f_1 but decreases with f_2. For sector 1, the increase in fixed

Figure 6: Tax revenue and gains from trade using the optimal corporate tax rates based in the closed economy formulas



production costs reduces the number of firms which reduces the "subsidy" amount given to this sector since its depreciation allowance rate is positive, therefore, corporate tax revenue from this sector increases. For sector 2, the explanation is analogous, but in reverse as this sector receives a negative depreciation allowance rate.

Lastly, I provide some examples of the welfare loses that government can incur by using the incorrect policy recommendation for the corporate tax instrument rates. For the open economy case, the policy recommendation under Pareto is to keep taxes unchanged when switching from autarky to trade. Thus, the "null" model will use the optimal tax rates found in the closed economy, for the log-normal assumption, and compute the open economy equilibrium. Then, I compare the outcomes from the "null" model with those of the "alternative" model where optimal tax rates change from their closed economy values. The welfare gains from using the correct taxes are found in the last row of Table 2. Governments can gain an additional 0.12%

to 0.32% in welfare by adjusting their corporate tax rates and, once more, the gains from using the correct tax rates increase with the degree of asymmetry across the sectors.

6. Empirical Evidence for using log-normal distributions

To finalize, I present some basic empirical findings suggesting log-normal distributions are a better fit for the empirical distribution of productivities for developing countries. The results of this section add to the evidence found by Sun et al. (2013) for Chinese firms, and Head et al. (2014) for French and Spanish firms.

I test the fitness of the Pareto distribution using multiple estimation methods on two different measure of productivity. The first "classic" measure, is the residual from the regression of firm output to inputs. This approach closely follows Del Gatto et al. (2006) since their paper has been cited multiple times to justify the validity of the Pareto distribution assumption for European firms. There are many issues involving the direct estimation of productivity which can be reduced if I were to use Olley-Pakes method; however, the data is not a proper panel. Therefore, the second productivity measure uses firms' sales. In this case, the identification assumption is not on the firms' technology but on the characteristic of the sector. Specifically, the assumption is that markets are monopolistic competitive with firms pricing their products at a markup.

Regardless of the productivity measure, the results strongly point in the direction of a log-normal distribution over Pareto to fit the empirical distribution of estimated productivities. Moreover, in most cases, the estimated Pareto distribution parameters violate the equilibrium conditions for the Melitz model, rendering it inapplicable.

6.1. Data

The firm-level data comes from the World Bank Enterprise Surveys database. The survey is given to firms with 5 or more full-time employees in 136 countries and contains a rich set of variables that provide a detailed picture of the firms' performance as well as the environment in which they operate. To ensure that data is comparable across countries, I make use of the standardized surveys for the period 2006 to 2013. These surveys were designed to be representative of the economy of each country, including its sector composition, with sample sizes that ensure robust statistical inferences.

I restrict the database to manufacturing firms that have completed the *manufacturing questionnaire*.[19] Observations are dropped if they are missing any of the following variables: total sales, net book value of

[19] There are 3 types of questionnaires in the survey: core, manufacturing, and service. The last two questionnaires contain the same questions as the core plus a set of further questions related to manufacturing or service sectors. Only the manufacturing questionnaire asks for the net book value of current machinery and equipment, which is our fixed capital variable.

machinery and equipment, and the number of full-time employees. Monetary variables are in nominal local currency units (LCU), so they converted into real values, in international 2010 dollars, using GDP deflators and PPP exchange rates from the World Bank's financial database. The measure of labor input is the number of full-time *permanent* workers employed during the fiscal year. A permanent full-time employee is a full time paid employee that has been in the firm for at least a year or have a renewal offer otherwise.[20]

The ISIC codes of the firms are used to classify them into 18 sectors. Table 3 shows the distribution of observations across sectors and geographical regions. The Middle East region (MNA) and the "Petroleum and Coal" sector were excluded from the final dataset due to an insufficient number of observations.

6.2. Testing the fitness of distributions: productivity as the residual of the production function

Assuming a Cobb-Douglas production, the estimated productivity of a firm j in sector s is $\exp(c_s + \epsilon_{j,s})$:

$$log(sales_j) = c_s + a_s log(K_j) + b_s log(N_j) + \epsilon_{j,s} \tag{6.1}$$

This regression is computed separately for each sector-region pair and table 5 reports the summary statistics. Eastern Europe and Central Asia region comes atop with an average (across sectors) of 222.62 while Africa stands last among all regions studied, with an alarming low 4.78.

Sectors inside each region are remarkably different reinforcing the point that corporate taxation models should explicitly consider sector heterogeneity. "Electric machinery" and "professional and scientific equipment" are the best performing sectors in all regions; however, there is no common worst performing sector(s).

6.2.a. *Pareto.–* I proceed to test if a Pareto distribution can properly fit the distribution of the estimated productivities. The functional form of the Pareto distribution implies that for a region r and sector s, the shape parameter k_s can be estimated by:

$$log(1 - F(x_{s,r})) = cons - k_s log(x_{s,r}) + \epsilon_{s,r}, \tag{6.2}$$

where F is the CDF of the distribution. Del Gatto et al. (2006) use the same estimation approach except for the inclusion of fixed year effect in the regression. Estimation results are found in Tables 6-10 under the OLS headings.

The OLS estimates for k_s are unreliable, but I report them for the sake of comparison with the values for

[20] A second measure that takes into account the *temporary* full-time workers yielded similar results.

Western Europe in Del Gatto et al. (2006). Most of the estimated k_s are below one which already presents a problem as the existence of an equilibrium in the Melitz model requires k_s to be greater than the elasticity of substitution minus one. Even though there is no consensus about the exact value of the Armington elasticity of substitution, the usual range is between 1 to 4.6; though there are estimates as high as 12 and as low as 0.51.[21] Thus, the elasticities bounds imply by the estimated k_s are plausible but not likely.

An alternative estimator for k_s has to be employed since the OLS estimator is biased, which is clear once 6.2 is re-written into:

$$log(1 - F(x_{s,r})) = k_{s,r}log(x_{min,s,r}) - k_s log(x_{s,r}) + \epsilon_{s,r},$$

the constant term in the previous regression is a function of the shape parameter and the lower bound of the support of $F(x)$. Due to the unreliability of the estimators of k_s using simple regression I use a maximum likelihood estimator instead; where I assume $x_{min,s,r}$ is equal to the minimum productivity observed in sector s in region r.[22]

Estimation using MLE generates a very different picture from that of OLS. First, the estimated shape parameters are smaller for all cases, which highlights the bias of the OLS estimator. The estimated distributions are not good approximations of the empirical distributions based on Kolmogorov-Smirnov (KS) tests.[23] Using a threshold of $p > 0.05$, there is no case but one in which the estimated Pareto distributions fit the data well. The "Professional and Scientific equipment" in the SAR region is the only case that passes the KS test; however, the number of observations is 19, which is below the $n = 50$ sample size required to ensure the asymptotic properties.[24]

I continue by testing if the Pareto distributions are a good fit for only a part of the empirical distributions for productivity. The new estimation procedure is based on Clauset et al. (2009). Step one: obtain the parameters of the estimated distribution using MLE on all observations. Next, compute the KS statistics and drop the smallest observation in the sample. Then, re-run the procedure from step one. The process continues until one of these conditions happen: the fitting of the estimated and empirical distribution worsens, or the deletion of observations generate a bias greater than 0.10.

Surprisingly, the new estimates yield no dramatic improvement with regards to the goodness of fit criteria as only two more cases passed the p-value threshold. Nonetheless, these cases are now a good fit without

[21] See Feenstra et al. (2018) for the most recent estimation of Armington elasticities.

[22] As a robustness check, the same estimation is carried assuming that $x_{min,s,r}$ is equal across all sectors in the same region, and its value is given by the smallest productivity observed in such region. Results of both estimations are almost the same. Furthermore, it can be shown that the MLE estimator for x_{min} is the minimum observed value from the sample.

[23] Kolmogorov-Smirnov tests the null hypothesis that the estimated distribution and the empirical distribution are statistically no different.

[24] This case was re-estimated using a finite sample bias correction. The new estimates are not substantially different from those reported in table 10.

discarding a significant amount of the empirical data.[25] What is clear, is that the shape parameters under these estimations are consistently higher than those obtained by setting x_{min} equal to the lowest value observed in the full sample of the sector-region pair. The values for k_s are closer to those found in Del Gatto et al. (2006) and other studies conducted in developed countries. Furthermore, if the upper bound for $\hat{x}_{min}$ is removed, then Pareto distributions are a decent approximation for the reduced data. This is a similar result to Head et al. (2014) who find that a Pareto distribution can approximate only the right tails of productivity distributions.

6.2.b. *Log-Normal.–* I follow by testing if log-normal distributions better describe the empirical distributions. The pdf of the log-normal distribution is:

$$f(x) = \left(\frac{1}{x\sqrt{2\pi v}}\right) exp\left(-\frac{(ln(x)-m)^2}{2v^2}\right)$$

where m, v are the scale and variance parameters. MLE is used to estimate these parameters and the results are reported in Tables 6-10.

The goodness of fit are a dramatic improvement over the Pareto distribution as attested by the Kolmogorov-Smirnov tests. Using the same *p-value* threshold of 0.05, the estimated log-normal distributions are a good fit for 72 out of 85 possible cases. Africa is the region with the least sectors (9) that are satisfactorily fitted while the rest of regions exhibit empirical productivity distributions that are well approximated for most, if not all, sectors.

The KS tests strongly suggest that the log-normal distribution describes the data successfully, but I perform an additional robustness check. I use Monte Carlo simulations to obtain reliable *p-values* that take into account the possibility that the KS statistic was a product of chance. Synthetic data is generated for each sector-region pair by drawing values from the estimated distribution that best fitted it. The number of draws equals the number of observations in the initial estimation. Then, the parameters to best fit the synthetic data are estimated and the KS statistic computed. The whole procedure is repeated 10000 times, for each sector-region pair, to obtain a precision of $\epsilon = 0.005$.[26] The *p-value* of the Monte Carlo simulation is the fraction of KS statistics larger than the one obtained in the original estimation. Higher *p-values* imply a lower probability that the results of the KS test were an outcome of chance.

Using a *p-value* threshold of $p > 0.05$ ($p > 0.10$) only 44 (38) sector-region pairs pass the Monte-Carlo simulation confirmation. This number of successful fits is lower than the amount based on the initial KS test

[25] Paper products in the EAP region drops 16% of observation, and Electric Machinery in LAC discards 7%.

[26] For computational considerations, the process is only carried for sector-regions that passed the initial KS test ($p > 0.05$).

criteria (72 cases).

6.3. Testing the fitness of distribution: sales data

The previous estimation using estimated values of firms' productivities is prone to many critics, especially regarding endogeneity issues between revenues and the amount of labor employed. Methods to solve this problem (such as Olley-Packes and its derivatives) require proper panel data which is not available in these surveys.

Therefore, I perform an alternative analysis that uses firms' revenues to infer the productivity parameter consistent with the model presented in this paper. The Melitz model implies that a firm with productivity φ has revenue:

$$r(\varphi) = p(\varphi)^{1-\sigma}\frac{Income}{\mathbb{P}^{1-\sigma}}$$
$$p(\varphi) = \frac{w}{\rho}\varphi^{-1}$$

Thus, revenues under this model have the same distributional form as φ since the transformation $Y = \varphi^{\sigma-1}$ preserves the shape of the distribution of φ. Specifically:

- If φ comes from a Pareto distribution with shape parameter k, then $\varphi^{\sigma-1} \sim Pareto(\tilde{k})$, where $\tilde{k} = \dfrac{k}{\sigma - 1}$
- If $\varphi \sim log\mathcal{N}(m, v)$ then $\varphi^{\sigma-1} \sim log\mathcal{N}((\sigma - 1)m, (\sigma - 1)v)$

The analysis using firms' revenues has additional advantages: it expands the number of non-missing observations significantly, and it can be used to test if the estimated parameters for the Pareto distribution satisfy the equilibrium conditions of the model. Previously, observations missing data for capital equipment were deleted since it was a necessary input to estimate the residual from the production function; however, for the current estimation method this is not necessary and thus valid observations increase by approximately 8000. Table 4 presents the distribution of valid observations across the sector and regions. [27]

Pareto or log-normal?

I conduct the same analysis as in section 6.2 and obtain similar findings for the fit of the Pareto distribution. Estimation results are found in Tables 11 to 15 with the first columns containing the estimated parameters for

[27] The analysis presented in the main body uses the full sample of firms. Nonetheless, concerns may arise since the sample has a mix of firms that sell only domestically with others that also engage in export. Therefore, separate analysis using: (i) firms whose revenues are from the domestic market exclusively, (ii) firms whose national sales account for 90 % or more of their revenue. The results are not significantly different from those using the full sample. In fact, when the sample consist of firms that only sell on the domestic market the conclusion in favor of using log-normal distributions to approximate the empirical distribution of productivity becomes stronger.

a Pareto distribution. Similarly to results using estimated productivities, the KS statistics for most sectors in each region are unfavorable to the hypothesis that revenues are Pareto distributed. Only 2 cases, out of a possible 85, pass the KS test with a threshold $p - value$ of 0.05. The modified MLE, in which the cutoff parameter is free to move, does not provide significant improvements except for "Electric Machinery" in LAC region which now passes the KS test by dropping only 7% of the lower observations.

Furthermore, the MLE results in values of $\tilde{k}$ that are below unity for all cases, which is problematic. The condition for the existence of an equilibrium in the Melitz model is $k > \sigma - 1 \implies \tilde{k} > 1$, therefore the estimated parameters using the Pareto distribution are inconsistent with this model. The modified MLE estimation barely improves the problem as it results in estimates of $\tilde{k}$ that are above one in most case but not by a significant amount. For Africa, the average $\tilde{k}$ remains below one, and the averages for the other regions are at most 1.66.

Finally, the estimated log-normal distributions perform remarkably well (and strongly outperform the Pareto distribution) in fitting the sales data, corroborating the first impressions from looking at the histograms of the logarithm of revenues. The log-normal distributions pass the Kolmogorov-Smirnov test for 70 sector-region pairs, out of a possible 85 cases, a dramatic improvement over the performance of the Pareto distribution. Once again, Monte Carlo simulations were performed (10 000 repetitions) to confirm the initial conclusions of the KS test. Using a p-value of 0.10 (0.05) the KS test is confirmed for 35 (42) cases, which is half of the cases that passed the initial KS test.

7. Conclusion

The question of the implication of assuming Pareto distributed productivities in a Melitz model has mostly been neglected until recently when Head et al. (2014), Bee and Schiavo (2018) showed their effects in equilibrium outcomes and the size of the gains from trade. However, the implications for policy of this de facto assumption have not been explored; specifically, the difference in properties of optimal corporate tax rates derived under the Pareto distribution and the log-normal distribution.

Using an enhanced Melitz model with heterogeneous sectors and a corporate taxation framework that resembles those observed in the real world, I demonstrated that using the Pareto distribution assumption mutes a transmission channel between the corporate tax rates and the equilibrium outcomes. Thus, I find not only quantitative differences between the optimal tax rates derived under the Pareto and log-normal distribution assumptions but also qualitative implications for the optimal corporate tax rates. Optimal rates derived under both distributional assumptions share many properties, especially the attribute that firms in sectors with higher elasticities of substitution get lower depreciation allowance rates on their fixed cost of productions. Quantitatively, the differences between the optimal rates derived under both distributions become more prominent with the degree of cross-sector heterogeneity.

Several policy implications arise from the qualitative differences between the optimal tax rates derived under each distributional assumption. First, the optimal corporate tax rates under the Pareto distribution assumption do not explicitly account for sector-specific fixed costs. In contrast, the optimal rates formulas derived under the log-normal assumption exploit sector heterogeneity along all dimensions. Thus, the government uses a more targeted approach by using the sector-specific depreciation allowance rates more heavily than under the policy implied in the Pareto setting. The inclusion of fixed costs in the determination of optimal rates seems to be critical as changes in these costs can be quite significant (for example, changes in entry costs following regulations targeting the competitiveness of the sector). There are also important implications in the context of the open economy. The model with symmetric countries shows that the probability of exporting is invariant to changes in tax rates when assuming Pareto distributions. This export tax neutrality translates into optimal corporate tax rates that do not change when a country moves from autarky to trade. Such property fails to hold in the log-normal case as the government must adjust their optimal corporate tax rates since their power to influence the equilibrium outcomes decreases when the country opens to trade.

Additionally, incorporating the corporate tax framework into the Melitz model provides a theoretical basis to explain conflicting empirical results regarding the relationship between corporate taxes and export dynamics. My model shows that decreasing the statutory corporate tax rate can increase or decrease the probability of becoming an exporter, the sign of this relationship depends on the level of the depreciation allowance rate on fixed costs. Nonetheless, increasing the depreciation allowance rate decreases the probability of exporting for all levels of the statutory corporate tax rate since this increase reduces the equilibrium productivity cutoff of domestic firms which makes them less competitive relative to firms in the other country.

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Table 1: Parameters and results for the different scenarios used to compute the inefficiencies from using the incorrect distribution for productivities. For outcomes with two values, the top comes from the "null" model while the value for the "alternative" model is directly underneath

Scenario	Almost Symmetric		Different Entry Cost		Different Cost of Production		More asymmetric Elasticities		Different Variance	
	Sector 1	Sector 2	Sector 1	Sector 2	Sector 1	Sector 2	Sector 1	Sector 2	Sector 1	Sector 2
Parameters										
Wage	1		1		1		1		1	
Labor Size	5		5		5		5		5	
q_0^G	0.5		0.5		0.5		0.5		0.5	
ψ	0.02		0.02		0.02		0.02		0.02	
Elasticity of Subs.	2.5	3	2.5	3	2.5	3	**1.5**	3	2.5	3
Share (α)	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
Fixed cost Production	0.1	0.1	0.1	0.1	0.1	**0.2**	0.1	0.1	0.1	0.1
Entry cost	0.5	0.5	0.5	**0.1**	0.5	0.5	0.5	0.5	0.5	0.5
m_i	2	2	2	2	2	2	2	2	2	**6**
v_i	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.7
k_i	3.12	3.12	3.12	3.12	3.12	3.12	3.12	3.12	3.12	2.61
φ_{min}	5.69	5.69	5.69	5.69	5.69	5.69	5.69	5.69	5.69	317.69
Results										
Number Firms	4.72 4.76	2.19 2.16	4.73 4.76	2.55 2.53	4.72 4.76	1.18 1.17	12.56 12.82	1.57 1.48	4.72 4.76	1.61 1.59
Sector Price Index	3.69 3.68	5.20 5.21	3.69 3.67	3.82 3.82	3.69 3.67	5.97 5.97	0.18 0.17	5.73 5.75	3.70 3.68	0.06 0.06
Depreciation Rate (%)	28.32 29.70	-28.32 -35.50	28.32 28.17	-28.32 -36.11	28.32 29.02	-28.32 -35.62	90.57 94.56	-90.57 -120.54	30.11 32.45	-25.11 -31.51
Corporate Tax (%)	30.71 30.31		30.71 29.91		30.71 30.13		40.15 35.85		31.25 31.06	
$\sum T_{alternative}$	0.5049		0.5083		0.5065		0.5696		0.5044	
$\mathbb{W}_{null}/\mathbb{W}_{alt}$	0.9986		0.9977		0.9982		0.9766		0.9987	

Table 2: Results for the open economy equilibrium with symmetric countries using the Pareto distribution recommend policy: the “optimal” corporate tax rates are the same as in the closed economy. The welfare gain from changing the corporate tax rates to their optimal value is given by $\mathbb{W}_{alternative}/\mathbb{W}_{null}$

Scenario	Almost Symmetric		Different Entry Cost		More asymmetric Elasticities		Different Variance	
	Pareto	Log-Normal	Pareto	Log-Normal	Pareto	Log-Normal	Pareto	Log-Normal
Sector 1								
$\%\Delta\varphi_{jj}$	16.436	9.567	16.436	9.550	8.349	10.283	16.436	9.599
$\tilde{\varphi}$	20.267	17.162	20.267	17.181	9.297	11.265	20.216	17.127
M	2.453	3.245	2.453	3.245	10.248	9.314	2.453	3.243
φ_{ex}	16.283	15.883	16.283	15.906	10.392	11.683	16.243	15.840
$\tilde{\varphi}_{ex}$	25.175	20.374	25.175	20.398	14.726	15.533	25.112	20.329
M_{ex}	1.245	1.498	1.245	1.496	2.433	3.339	1.245	1.500
GFT($\%\Delta\tilde{\varphi}_{tot}$)	21.607	9.801	21.607	9.794	16.824	12.671	21.607	9.815
% decrease in Prices	16.436	9.555	16.436	9.531	8.349	9.674	16.436	9.579
Sector 2								
$\%\Delta\varphi_{jj}$	18.703	8.510	18.704	6.379	18.704	7.987	24.595	12.585
$\tilde{\varphi}$	42.955	21.059	71.879	26.645	46.187	22.148	217.370	38.734
M	0.534	1.467	0.534	1.766	0.367	1.006	0.105	1.043
φ_{ex}	26.716	18.931	44.704	25.240	28.726	20.173	71.630	31.110
$\tilde{\varphi}_{ex}$	50.825	24.115	85.048	30.792	54.649	25.422	257.196	44.296
M_{ex}	0.315	0.718	0.315	0.729	0.217	0.475	0.068	0.598
GFT($\%\Delta\tilde{\varphi}_{tot}$)	22.155	8.003	22.155	6.934	22.155	7.789	28.415	11.193
% decrease in Prices	18.703	8.503	18.704	6.368	18.704	7.868	24.595	12.573
Country								
Tax Collected	0.500	0.499	0.500	0.499	0.500	0.486	0.500	0.499
Welfare	77.430	64.804	99.428	74.543	305.773	275.690	125.039	82.248
Gains from Trade	16.901	8.632	17.048	7.624	13.284	8.383	19.956	10.665
% ($\mathbb{W}_{alt}/\mathbb{W}_{null} - 1$)		0.12		0.163		0.327		0.154

Table 3: Distribution of observations across sectors and regions

	Region						
	AFR	EAP	ECA	LAC	MNA	SAR	Total
Food beverages and tobacco	1,532	402	1,130	2,195	211	549	6,019
Textiles	185	326	287	872	7	484	2,161
Wearing apparel except footwear	971	345	611	1,260	33	452	3,672
Leather products and footwear	111	42	59	263	3	357	835
Wood products except furniture	232	61	244	145	15	66	763
Paper products	70	38	68	62	6	40	284
Printing and Publishing	226	56	214	194	10	68	768
Petroleum and Coal	5	7	6	8	6	2	34
Chemicals	336	276	286	1,323	40	283	2,544
Rubber and plastic	177	314	195	546	40	109	1,381
Other non-metallic products	207	374	324	391	172	94	1,562
Metallic products	89	101	55	126	6	85	462
Fabricated metal products	499	248	604	895	47	75	2,368
Machinery except electrical	112	173	431	622	9	78	1,425
Electric machinery	61	159	165	144	6	70	605
Professional and scientific equipment	19	82	107	73	2	15	298
Transport equipment	48	128	64	134	2	33	409
other manufacturing	717	106	327	453	39	143	1,785
Total	5,597	3,238	5,177	9,706	654	3,003	27,375

Table 4: Distribution of non-missing observations, across sectors and regions, for the analysis using firms' revenues.

	Region					
	AFR	EAP	ECA	LAC	SAR	Total
Food beverages and tobacco	1,936	553	1,684	2,793	706	7,672
Textiles	272	418	387	1,120	639	2,836
Wearing apparel except footwear	1,199	458	899	1,645	506	4,707
Leather products and footwear	143	55	81	306	386	971
Wood products except furniture	324	97	360	186	103	1,070
Paper products	88	56	95	96	70	405
Printing and Publishing	318	71	347	261	77	1,074
Chemicals	418	380	413	1,582	333	3,126
Rubber and plastic	213	418	326	651	141	1,749
Other non-metallic products	284	522	591	540	133	2,070
Metallic products	125	125	90	156	159	655
Fabricated metal products	654	287	850	1,083	88	2,962
Machinery except electrical	142	188	698	748	112	1,888
Electric machinery	73	215	257	175	71	791
Professional and scientific equipment	21	109	180	81	15	406
Transport equipment	64	158	93	167	55	537
other manufacturing	1,032	148	504	575	195	2,454
Total	7,306	4,258	7,855	12,165	3,789	35,373

Table 5: Summary Statistics for the estimate productivities. The means are in hundreds of 2010 International Dollars

	AFR			EAP			ECA			LAC			SAR		
	Mean	σ	Obs.	Mean	σ	Obs.	Mean	σ	Obs.	Mean	σ	Obs.	Mean	σ	Obs.
Food beverages and tobacco	1.55	5.23	1623	5.95	9.42	403	122.5	203.11	1112	23.4	28.56	2172	15.3	38.48	542
Textiles	3.71	10.93	187	5.76	6.62	329	55.32	87.72	284	43.57	42.81	872	17.53	23.24	481
Wearing apparel except footwear	0.66	1.29	1008	20.79	29.24	343	31.96	51.02	601	37.13	41.8	1251	15.29	18.54	448
Leather products and footwear	3.87	7.57	112	21.89	22.83	42	129.24	859.62	59	7.61	6.68	268	12.36	15.48	352
Wood products except furniture	4.23	19.74	240	7.75	11.54	63	105.21	156.87	240	26.93	47.4	143	26.35	43.91	66
Paper products	16.04	39.23	72	8.93	6.74	38	8803.15	47086.7	68	56.6	54.26	62	24.75	33.99	40
Printing and Publishing	0.45	0.8	234	28.07	46.6	56	36.41	75.76	210	184.31	251.72	192	6.89	9.76	68
Chemicals	9.82	31.81	343	18.6	37.73	272	202.89	325.77	284	73.7	86.32	1306	11.97	18.44	279
Rubber and plastic	4.38	13.21	187	66.52	97.62	311	49.36	57.97	193	54.71	39.86	537	5.68	8.1	108
Other non-metallic products	4.94	27.06	215	11.61	19.31	372	74.92	97.92	320	17.42	34.29	388	213.41	362.35	95
Metallic products	16.53	40.68	91	17.62	25.01	99	161.27	534.39	55	17.52	35.56	125	55.78	72.53	85
Fabricated metal products	0.95	2.61	530	32.44	87.44	246	50.73	66.67	594	53.18	55.13	885	14.24	23.62	76
Machinery except electrical	5.83	24.95	124	23.39	38.69	171	267.15	489.27	423	45.49	47.66	620	24.93	38.71	78
Electric machinery	22.76	115.08	63	110.05	165.49	157	115.01	149.2	163	23.05	20.09	142	21.81	80	70
Professional and scientific equip	195.21	277.3	19	55.36	77	82	305.64	412.9	106	144.52	135.2	73	194.1	179.31	15
Transport equip	26.87	33.11	48	58.56	52.58	126	86.75	127.47	64	13.07	31.25	138	21.09	84.54	34
other manufacturing	9.16	45.94	765	12.19	14.42	104	78.03	142.34	324	7.19	8.73	463	56.93	57.57	142
Total	4.78	31.28	5861	27.68	64.97	3214	222.62	5494.41	5100	42.43	67.06	9637	25.63	82.84	2979

Table 6: Africa: Parameter estimation and goodness of fit for the Pareto and log-normal distributions. Empirical distribution of productivities based on estimation of the residual from a Cobb-Douglas production technology

		OLS		MLE			MLE mod				Log-normal			
	Obs.	k_s	R^2	k_s	x_{min}	K.S p-value	k_s	x_{min}	K.S p-value	ratio of $x < x_{min}$	m	v	K.S p-value	Monte Carlo p-value
Food beverages and tobacco	1623	0.63	0.9	0.26	0.89	0.00	0.96	98.75	0.00	0.74	3.66	1.49	0.00	
Textiles	187	0.76	0.93	0.37	8.31	0.00	1.04	145.54	0.00	0.57	4.83	1.24	0.39	0.052
Wearing apparel except footwear	1008	0.71	0.88	0.34	1.32	0.00	0.96	31.85	0.00	0.58	3.25	1.31	0.04	
Leather products and footwear	112	0.6	0.84	0.27	3.50	0.00	0.79	97.97	0.00	0.42	4.89	1.47	0.81	0.439
Wood products except furniture	240	0.58	0.91	0.25	1.25	0.00	0.77	174.50	0.00	0.78	4.16	1.62	0.05	0.000
Paper products	72	0.67	0.9	0.30	17.78	0.00	0.88	646.76	0.00	0.65	6.23	1.35	0.27	0.017
Printing and Publishing	234	0.73	0.9	0.36	1.18	0.00	0.95	18.73	0.00	0.52	2.92	1.26	0.36	0.044
Chemicals	343	0.65	0.95	0.31	8.91	0.00	0.72	128.87	0.00	0.40	5.38	1.48	0.01	
Rubber and plastic	187	0.65	0.95	0.33	4.63	0.00	0.72	54.36	0.00	0.37	4.61	1.47	0.00	
Other non-metallic products	215	0.62	0.94	0.29	1.90	0.00	0.70	25.96	0.00	0.25	4.14	1.54	0.03	
Metallic products	91	0.55	0.75	0.15	0.63	0.00	0.89	352.68	0.00	0.33	6.29	1.49	0.29	0.024
Fabricated metal products	530	0.66	0.9	0.30	1.00	0.00	0.86	37.15	0.00	0.62	3.33	1.43	0.14	0.003
Machinery except electrical	124	0.55	0.93	0.28	2.24	0.00	0.61	28.09	0.00	0.24	4.39	1.70	0.02	
Electric machinery	63	0.5	0.86	0.17	0.44	0.00	0.64	48.07	0.01	0.17	4.96	1.85	0.22	0.012
Professional and scientific equip.	19	0.58	0.78	0.35	534.23	0.04	1.17	10564.04	0.00	0.47	9.12	1.31	0.99	0.955
Transport equip.	48	0.86	0.89	0.47	186.02	0.00	0.89	695.75	0.01	0.19	7.36	1.02	0.87	0.553
Other manufacturing	765	0.58	0.91	0.20	0.78	0.00	0.69	82.49	0.00	0.42	4.86	1.64	0.00	
Average		0.64	0.89	0.29	45.59		0.84	778.33			4.96	1.45		

Table 7: East Asia Pacific: Parameter estimation and goodness of fit for the Pareto and log-normal distributions. Empirical distribution of productivities based on estimation of the residual from a Cobb-Douglas production technology

		OLS		MLE			MLE mod				Log-normal			
	Obs.	k_s	R^2	k_s	x_{min}	K.S p-value	k_s	x_{min}	K.S p-value	ratio of $x < x_{min}$	m	v	K.S p-value	Monte Carlo p-value
Food beverages and tobacco	403	0.87	0.88	0.27	7.84	0.00	1.29	412.73	0.00	0.61	5.76	1.06	0.46	0.092
Textiles	329	0.96	0.82	0.37	25.57	0.00	1.44	353.29	0.00	0.46	5.93	0.92	0.26	0.017
Wearing apparel except footwear	343	0.97	0.86	0.36	79.90	0.00	1.47	1360.27	0.00	0.53	7.15	0.93	0.11	0.002
Leather products and footwear	42	1.16	0.87	0.60	301.09	0.00	1.37	1036.87	0.00	0.26	7.37	0.76	0.80	0.426
Wood products except furniture	63	0.77	0.79	0.35	25.47	0.00	1.33	367.45	0.00	0.37	6.06	1.08	0.33	0.029
Paper products	38	1.29	0.86	0.67	161.08	0.00	1.47	430.78	0.13	0.16	6.57	0.66	0.77	0.377
Printing and Publishing	56	0.9	0.91	0.47	181.12	0.00	1.35	1621.48	0.00	0.52	7.33	1.01	0.82	0.461
Chemicals	272	0.91	0.87	0.42	89.79	0.00	1.60	2001.69	0.00	0.76	6.90	1.02	0.55	0.148
Rubber and plastic	311	0.92	0.87	0.38	286.16	0.00	1.39	5351.66	0.00	0.66	8.26	0.99	0.45	0.076
Other non-metallic products	372	0.96	0.87	0.35	39.35	0.00	1.22	550.88	0.00	0.42	6.52	0.96	0.22	0.010
Metallic products	99	1.04	0.92	0.62	219.95	0.00	1.52	1513.42	0.00	0.66	7.00	0.89	0.81	0.451
Fabricated metal products	246	0.96	0.9	0.44	173.30	0.00	1.24	1480.11	0.00	0.47	7.41	0.98	0.19	0.007
Machinery except electrical	171	0.96	0.92	0.50	176.33	0.00	1.21	1164.73	0.00	0.48	7.17	0.98	0.45	0.081
Electric machinery	157	0.87	0.87	0.32	276.31	0.00	1.31	5890.42	0.00	0.47	8.71	1.03	0.14	0.003
Professional and scientific equipment	82	0.86	0.88	0.44	313.63	0.00	1.21	2802.60	0.00	0.44	8.04	1.04	0.49	0.100
Transport equipment	126	1.12	0.85	0.52	630.69	0.00	1.49	3914.81	0.00	0.46	8.36	0.79	0.65	0.228
other manufacturing	104	0.92	0.84	0.48	97.41	0.00	1.44	818.60	0.00	0.50	6.65	0.95	0.94	0.738
Average		0.97	0.87	0.45	181.47		1.37	1827.75			7.13	0.94		

Table 8: East Europe & Central Asia: Parameter estimation and goodness of fit for the Pareto and log-normal distributions. Empirical distribution of productivities based on estimation of the residual from a Cobb-Douglas production technology

		OLS		MLE			MLE mod				Log-normal			
	Obs.	k_s	R^2	k_s	x_{min}	K.S p-value	k_s	x_{min}	K.S p-value	ratio of $x < x_{min}$	m	v	K.S p-value	Monte Carlo p-value
Food beverages and tobacco	1112	0.78	0.85	0.33	278.38	0.00	1.17	8683.76	0.00	0.65	8.69	1.17	0.19	0.007
Textiles	284	0.79	0.82	0.35	170.75	0.00	1.38	3966.73	0.00	0.58	7.99	1.12	0.78	0.403
Wearing apparel except footwear	601	0.95	0.9	0.42	169.76	0.00	1.31	2574.45	0.00	0.68	7.49	0.99	0.09	0.001
Leather products and footwear	59	0.88	0.95	0.45	110.01	0.00	0.94	460.45	0.01	0.19	6.91	1.30	0.02	
Wood products except furniture	240	0.87	0.9	0.44	563.66	0.00	0.99	3510.19	0.00	0.34	8.62	1.07	0.16	0.004
Paper products	68	0.67	0.8	0.26	3438.73	0.00	1.00	116026.90	0.00	0.38	11.94	1.38	0.26	0.016
Printing and Publishing	210	0.88	0.88	0.40	154.65	0.00	1.36	2775.91	0.00	0.67	7.54	1.04	0.62	0.194
Chemicals	284	0.79	0.81	0.29	330.44	0.00	1.44	14939.70	0.00	0.60	9.29	1.11	0.64	0.213
Rubber and plastic	193	0.9	0.78	0.34	161.90	0.00	1.43	3045.54	0.00	0.45	8.07	0.95	0.63	0.208
Other non-metallic products	320	0.76	0.81	0.31	155.42	0.00	1.33	5973.27	0.00	0.62	8.29	1.15	0.99	0.929
Metallic products	55	0.65	0.92	0.31	144.95	0.00	0.78	2364.67	0.00	0.38	8.23	1.44	0.19	0.006
Fabricated metal products	594	0.94	0.85	0.32	139.47	0.00	1.36	3627.29	0.00	0.59	8.03	0.97	0.12	0.002
Machinery except electrical	423	0.89	0.86	0.38	1070.20	0.00	1.49	33762.05	0.00	0.81	9.58	1.03	0.35	0.044
Electric machinery	163	0.88	0.84	0.36	426.96	0.00	1.39	8455.83	0.00	0.60	8.83	1.00	0.81	0.452
Professional and scientific equipment	106	0.81	0.86	0.41	1435.32	0.00	1.41	25665.57	0.00	0.63	9.71	1.10	0.66	0.236
Transport equipment	64	0.83	0.74	0.36	317.55	0.00	1.57	5243.98	0.00	0.42	8.57	0.98	0.74	0.336
other manufacturing	324	0.82	0.89	0.35	220.82	0.00	1.05	3104.72	0.00	0.44	8.22	1.13	0.14	0.002
Average		0.83	0.85	0.36	546.41		1.26	14363.59			8.59	1.11		

Table 9: Latin America and the Caribbean: Parameter estimation and goodness of fit for the Pareto and log-normal distributions. Empirical distribution of productivities based on estimation of the residual from a Cobb-Douglas production technology

		OLS		MLE			MLE mod				Log-normal			
	Obs.	k_s	R^2	k_s	x_{min}	K.S p-value	k_s	x_{min}	K.S p-value	ratio of $x < x_{min}$	m	v	K.S p-value	Monte Carlo p-value
Food beverages and tobacco	2172	0.98	0.83	0.32	63.62	0.00	1.81	3743.14	0.00	0.85	7.32	0.92	0.05	
Textiles	872	0.91	0.74	0.27	74.84	0.00	1.84	4770.79	0.00	0.70	7.99	0.94	0.04	
Wearing apparel except footwear	1251	0.95	0.81	0.33	121.37	0.00	1.69	4004.55	0.00	0.72	7.79	0.93	0.43	0.071
Leather products and footwear	268	0.92	0.7	0.25	10.31	0.00	1.91	704.29	0.00	0.59	6.30	0.88	0.15	0.005
Wood products except furniture	143	1.03	0.87	0.46	186.39	0.00	1.66	2017.19	0.00	0.57	7.41	0.89	0.73	0.332
Paper products	62	1	0.71	0.43	405.45	0.00	1.22	2326.14	0.02	0.16	8.34	0.80	0.74	0.329
Printing and Publishing	192	1.29	0.8	0.38	988.92	0.00	2.06	12703.50	0.00	0.41	9.53	0.70	0.06	0.002
Chemicals	1306	1.01	0.82	0.31	194.52	0.00	2.01	11126.73	0.00	0.82	8.50	0.89	0.27	0.021
Rubber and plastic	537	1.24	0.79	0.46	481.08	0.00	2.35	5880.26	0.00	0.65	8.37	0.70	0.80	0.434
Other non-metallic products	388	0.97	0.88	0.37	65.87	0.00	1.34	884.71	0.00	0.45	6.89	0.95	0.14	0.003
Metallic products	125	0.89	0.82	0.37	63.97	0.00	1.27	931.45	0.00	0.46	6.88	1.00	0.98	0.898
Fabricated metal products	885	1.11	0.84	0.37	260.83	0.00	1.65	4216.79	0.00	0.56	8.24	0.81	0.37	0.045
Machinery except electrical	620	0.97	0.78	0.32	138.30	0.00	2.02	5270.53	0.00	0.72	8.04	0.89	0.54	0.132
Electric machinery	142	1.12	0.85	0.46	188.51	0.00	0.99	680.91	0.28	0.07	7.43	0.79	0.18	0.006
Professional and scientific equipment	73	1.08	0.85	0.53	1543.51	0.00	1.27	5755.48	0.03	0.15	9.25	0.80	0.09	0.001
Transport equipment	138	0.67	0.64	0.21	6.04	0.00	1.26	525.59	0.00	0.33	6.51	1.17	0.03	
other manufacturing	463	0.83	0.76	0.24	6.90	0.00	1.84	988.21	0.00	0.79	6.09	1.03	0.31	0.032
Average		0.99	0.79	0.36	282.38		1.66	3913.54			7.70	0.89		

Table 10: South Asia: Parameter estimation and goodness of fit for the Pareto and log-normal distributions. Empirical distribution of productivities based on estimation of the residual from a Cobb-Douglas production technology

		OLS		MLE			MLE mod				Log-normal			
	Obs.	k_s	R^2	k_s	x_{min}	K.S p-value	k_s	x_{min}	K.S p-value	ratio of $x < x_{min}$	m	v	K.S p-value	Monte Carlo p-value
Food beverages and tobacco	542	0.77	0.91	0.38	43.79	0.00	1.24	1569.81	0.00	0.78	6.42	1.21	0.10	0.001
Textiles	481	0.94	0.87	0.45	113.92	0.00	1.67	2221.36	0.00	0.77	6.97	0.97	0.37	0.045
Wearing apparel except footwear	448	1.04	0.88	0.49	128.15	0.00	1.80	2190.74	0.00	0.81	6.90	0.89	0.55	0.142
Leather products and footwear	352	0.86	0.83	0.37	50.71	0.00	1.71	1529.58	0.00	0.75	6.59	1.03	0.98	0.871
Wood products except furniture	66	0.97	0.83	0.55	259.30	0.00	1.18	1248.29	0.00	0.41	7.37	0.92	0.65	0.224
Paper products	40	0.76	0.91	0.43	121.32	0.00	0.91	728.25	0.00	0.33	7.13	1.13	0.47	0.082
Printing and Publishing	68	1.22	0.89	0.48	59.13	0.00	1.59	394.49	0.00	0.40	6.16	0.77	0.62	0.200
Chemicals	279	0.96	0.85	0.47	85.67	0.00	1.67	1212.45	0.00	0.70	6.59	0.95	0.93	0.709
Rubber and plastic	108	0.74	0.85	0.39	21.90	0.00	0.69	93.68	0.00	0.17	5.65	1.18	0.86	0.543
Other non-metallic products	95	0.72	0.74	0.19	53.87	0.00	1.38	16824.12	0.00	0.63	9.33	1.15	0.88	0.589
Metallic products	85	0.86	0.87	0.46	361.42	0.00	1.12	2496.12	0.00	0.39	8.07	1.03	0.93	0.730
Fabricated metal products	76	0.89	0.83	0.45	86.49	0.00	1.51	959.18	0.00	0.58	6.70	0.98	0.49	0.102
Machinery except electrical	78	1.08	0.83	0.46	183.44	0.00	1.21	988.14	0.00	0.23	7.41	0.83	0.95	0.775
Electric machinery	70	0.9	0.92	0.52	113.83	0.00	0.84	313.88	0.01	0.19	6.65	1.10	0.20	0.008
Professional and scientific equipment	15	1.08	0.9	1.06	5494.80	0.83	1.06	5494.80	0.83	0.00	9.56	0.77	0.83	0.473
Transport equipment	34	0.7	0.9	0.46	41.97	0.07	1.11	485.10	0.00	0.53	5.92	1.41	0.78	0.399
other manufacturing	142	0.95	0.82	0.41	323.44	0.00	1.06	1977.84	0.00	0.18	8.24	0.91	0.81	0.435
Average		0.91	0.86	0.47	443.71		1.28	2395.76			7.16	1.01		

Table 11: Africa: Parameter estimation and goodness of fit for the Pareto and log-normal distributions. Empirical distribution of productivities using a transformation on firms' revenues.

		MLE			MLE mod				Log-normal			
	Obs.	k_s	x_{min}	K.S p-value	k_s	x_{min}	K.S p-value	ratio of $x < x_{min}$	m	v	K.S p-value	Monte Carlo p-value
Food beverages and tobacco	1623	0.26	0.89	0.00	0.96	98.75	0.00	0.74	3.66	1.49	0.00	
Textiles	187	0.37	8.31	0.00	1.04	145.54	0.00	0.57	4.83	1.24	0.39	0.541
Wearing apparel except footwear	1008	0.34	1.32	0.00	0.96	31.85	0.00	0.58	3.25	1.31	0.04	
Leather products and footwear	112	0.27	3.50	0.00	0.79	97.97	0.00	0.42	4.89	1.47	0.81	0.081
Wood products except furniture	240	0.25	1.25	0.00	0.77	174.50	0.00	0.78	4.16	1.62	0.05	0.003
Paper products	72	0.30	17.78	0.00	0.88	646.76	0.00	0.65	6.23	1.35	0.27	0.097
Printing and Publishing	234	0.36	1.18	0.00	0.95	18.73	0.00	0.52	2.92	1.26	0.36	0.005
Chemicals	343	0.31	8.91	0.00	0.72	128.87	0.00	0.40	5.38	1.48	0.01	0.038
Rubber and plastic	187	0.33	4.63	0.00	0.72	54.36	0.00	0.37	4.61	1.47	0.00	0.167
Other non-metallic products	215	0.29	1.90	0.00	0.70	25.96	0.00	0.25	4.14	1.54	0.03	0.021
Metallic products	91	0.15	0.63	0.00	0.89	352.68	0.00	0.33	6.29	1.49	0.29	0.017
Fabricated metal products	530	0.30	1.00	0.00	0.86	37.15	0.00	0.62	3.33	1.43	0.14	0.002
Machinery except electrical	124	0.28	2.24	0.00	0.61	28.09	0.00	0.24	4.39	1.70	0.02	0.114
Electric machinery	63	0.17	0.44	0.00	0.64	48.07	0.01	0.17	4.96	1.85	0.22	0.775
Professional and scientific equip.	19	0.35	534.23	0.04	1.17	10564.04	0.00	0.47	9.12	1.31	0.99	0.323
Transport equip.	48	0.47	186.02	0.00	0.89	695.75	0.01	0.19	7.36	1.02	0.87	0.852
Other manufacturing	765	0.20	0.78	0.00	0.69	82.49	0.00	0.42	4.86	1.64	0.00	
Average		0.29	45.59		0.84	778.33		0.45	4.96	1.45		

Notes. The values for x_{min} have been divided by 1000

Table 12: East Asia Pacific: Parameter estimation and goodness of fit for the Pareto and log-normal distributions. Empirical distribution of productivities using a transformation on firms' revenues.

		MLE			MLE mod				Log-normal			
	Obs.	k_s	x_{min}	K.S p-value	k_s	x_{min}	K.S p-value	ratio of $x < x_{min}$	m	v	K.S p-value	Monte Carlo p-value
Food beverages and tobacco	403	0.27	7.84	0.00	1.29	412.73	0.00	0.61	5.76	1.06	0.46	0.029
Textiles	329	0.37	25.57	0.00	1.44	353.29	0.00	0.46	5.93	0.92	0.26	
Wearing apparel except footwear	343	0.36	79.90	0.00	1.47	1360.27	0.00	0.53	7.15	0.93	0.11	0.001
Leather products and footwear	42	0.60	301.09	0.00	1.37	1036.87	0.00	0.26	7.37	0.76	0.80	0.078
Wood products except furniture	63	0.35	25.47	0.00	1.33	367.45	0.00	0.37	6.06	1.08	0.33	0.035
Paper products	38	0.67	161.08	0.00	1.47	430.78	0.13	0.16	6.57	0.66	0.77	0.239
Printing and Publishing	56	0.47	181.12	0.00	1.35	1621.48	0.00	0.52	7.33	1.01	0.82	0.003
Chemicals	272	0.42	89.79	0.00	1.60	2001.69	0.00	0.76	6.90	1.02	0.55	
Rubber and plastic	311	0.38	286.16	0.00	1.39	5351.66	0.00	0.66	8.26	0.99	0.45	
Other non-metallic products	372	0.35	39.35	0.00	1.22	550.88	0.00	0.42	6.52	0.96	0.22	0.001
Metallic products	99	0.62	219.95	0.00	1.52	1513.42	0.00	0.66	7.00	0.89	0.81	0.507
Fabricated metal products	246	0.44	173.30	0.00	1.24	1480.11	0.00	0.47	7.41	0.98	0.19	0.002
Machinery except electrical	171	0.50	176.33	0.00	1.21	1164.73	0.00	0.48	7.17	0.98	0.45	0.323
Electric machinery	157	0.32	276.31	0.00	1.31	5890.42	0.00	0.47	8.71	1.03	0.14	0.381
Professional and scientific equipment	82	0.44	313.63	0.00	1.21	2802.60	0.00	0.44	8.04	1.04	0.49	0.160
Transport equipment	126	0.52	630.69	0.00	1.49	3914.81	0.00	0.46	8.36	0.79	0.65	0.034
other manufacturing	104	0.48	97.41	0.00	1.44	818.60	0.00	0.50	6.65	0.95	0.94	0.135
Average		0.45	181.47		1.37	1827.75			7.13	0.94		

Notes. The values for x_{min} have been divided by 1000

Table 13: Eastern Europe & Central Asia region: Parameter estimation and goodness of fit for the Pareto and log-normal distributions. Empirical distribution of productivities using a transformation on firms' revenues.

		MLE			MLE mod				Log-normal			
	Obs.	k_s	x_{min}	K.S p-value	k_s	x_{min}	K.S p-value	ratio of $x < x_{min}$	m	v	K.S p-value	Monte Carlo p-value
Food beverages and tobacco	1112	0.33	278.38	0.00	1.17	8683.76	0.00	0.65	8.69	1.17	0.19	
Textiles	284	0.35	170.75	0.00	1.38	3966.73	0.00	0.58	7.99	1.12	0.78	
Wearing apparel except footwear	601	0.42	169.76	0.00	1.31	2574.45	0.00	0.68	7.49	0.99	0.09	0.414
Leather products and footwear	59	0.45	110.01	0.00	0.94	460.45	0.01	0.19	6.91	1.30	0.02	0.574
Wood products except furniture	240	0.44	563.66	0.00	0.99	3510.19	0.00	0.34	8.62	1.07	0.16	0.460
Paper products	68	0.26	3438.73	0.00	1.00	116026.90	0.00	0.38	11.94	1.38	0.26	0.895
Printing and Publishing	210	0.40	154.65	0.00	1.36	2775.91	0.00	0.67	7.54	1.04	0.62	0.239
Chemicals	284	0.29	330.44	0.00	1.44	14939.70	0.00	0.60	9.29	1.11	0.64	0.048
Rubber and plastic	193	0.34	161.90	0.00	1.43	3045.54	0.00	0.45	8.07	0.95	0.63	0.011
Other non-metallic products	320	0.31	155.42	0.00	1.33	5973.27	0.00	0.62	8.29	1.15	0.99	0.310
Metallic products	55	0.31	144.95	0.00	0.78	2364.67	0.00	0.38	8.23	1.44	0.19	0.096
Fabricated metal products	594	0.32	139.47	0.00	1.36	3627.29	0.00	0.59	8.03	0.97	0.12	0.180
Machinery except electrical	423	0.38	1070.20	0.00	1.49	33762.05	0.00	0.81	9.58	1.03	0.35	0.041
Electric machinery	163	0.36	426.96	0.00	1.39	8455.83	0.00	0.60	8.83	1.00	0.81	0.001
Professional and scientific equipment	106	0.41	1435.32	0.00	1.41	25665.57	0.00	0.63	9.71	1.10	0.66	0.302
Transport equipment	64	0.36	317.55	0.00	1.57	5243.98	0.00	0.42	8.57	0.98	0.74	0.596
other manufacturing	324	0.35	220.82	0.00	1.05	3104.72	0.00	0.44	8.22	1.13	0.14	0.065
Average		0.36	546.41		1.26	14363.59			8.59	1.11		

Notes. The values for x_{min} have been divided by 1000

Table 14: Latin America and the Caribbean: Parameter estimation and goodness of fit for the Pareto and log-normal distributions. Empirical distribution of productivities using a transformation on firms' revenues.

		MLE			MLE mod				Log-normal			
	Obs.	k_s	x_{min}	K.S p-value	k_s	x_{min}	K.S p-value	ratio of $x < x_{min}$	m	v	K.S p-value	Monte Carlo p-value
Food beverages and tobacco	2172	0.32	63.62	0.00	1.81	3743.14	0.00	0.85	7.32	0.92	0.05	
Textiles	872	0.27	74.84	0.00	1.84	4770.79	0.00	0.70	7.99	0.94	0.04	0.057
Wearing apparel except footwear	1251	0.33	121.37	0.00	1.69	4004.55	0.00	0.72	7.79	0.93	0.43	0.046
Leather products and footwear	268	0.25	10.31	0.00	1.91	704.29	0.00	0.59	6.30	0.88	0.15	0.342
Wood products except furniture	143	0.46	186.39	0.00	1.66	2017.19	0.00	0.57	7.41	0.89	0.73	0.473
Paper products	62	0.43	405.45	0.00	1.22	2326.14	0.02	0.16	8.34	0.80	0.74	0.006
Printing and Publishing	192	0.38	988.92	0.00	2.06	12703.50	0.00	0.41	9.53	0.70	0.06	0.025
Chemicals	1306	0.31	194.52	0.00	2.01	11126.73	0.00	0.82	8.50	0.89	0.27	
Rubber and plastic	537	0.46	481.08	0.00	2.35	5880.26	0.00	0.65	8.37	0.70	0.80	0.000
Other non-metallic products	388	0.37	65.87	0.00	1.34	884.71	0.00	0.45	6.89	0.95	0.14	0.000
Metallic products	125	0.37	63.97	0.00	1.27	931.45	0.00	0.46	6.88	1.00	0.98	0.149
Fabricated metal products	885	0.37	260.83	0.00	1.65	4216.79	0.00	0.56	8.24	0.81	0.37	0.004
Machinery except electrical	620	0.32	138.30	0.00	2.02	5270.53	0.00	0.72	8.04	0.89	0.54	0.151
Electric machinery	142	0.46	188.51	0.00	0.99	680.91	0.28	0.07	7.43	0.79	0.18	0.125
Professional and scientific equipment	73	0.53	1543.51	0.00	1.27	5755.48	0.03	0.15	9.25	0.80	0.09	0.009
Transport equipment	138	0.21	6.04	0.00	1.26	525.59	0.00	0.33	6.51	1.17	0.03	0.555
other manufacturing	463	0.24	6.90	0.00	1.84	988.21	0.00	0.79	6.09	1.03	0.31	0.151
Average		0.36	282.38		1.66	3913.54			7.70	0.89		

Notes. The values for x_{min} have been divided by 1000

Table 15: South Asia Region: Parameter estimation and goodness of fit for the Pareto and log-normal distributions. Empirical distribution of productivities using a transformation on firms' revenues.

		MLE			MLE mod				Log-normal			
	Obs.	k_s	x_{min}	K.S p-value	k_s	x_{min}	K.S p-value	ratio of $x < x_{min}$	m	v	K.S p-value	Monte Carlo p-value
Food beverages and tobacco	542	0.38	43.79	0.00	1.24	1569.81	0.00	0.78	6.42	1.21	0.10	
Textiles	481	0.45	113.92	0.00	1.67	2221.36	0.00	0.77	6.97	0.97	0.37	
Wearing apparel except footwear	448	0.49	128.15	0.00	1.80	2190.74	0.00	0.81	6.90	0.89	0.55	
Leather products and footwear	352	0.37	50.71	0.00	1.71	1529.58	0.00	0.75	6.59	1.03	0.98	
Wood products except furniture	66	0.55	259.30	0.00	1.18	1248.29	0.00	0.41	7.37	0.92	0.65	0.014
Paper products	40	0.43	121.32	0.00	0.91	728.25	0.00	0.33	7.13	1.13	0.47	0.424
Printing and Publishing	68	0.48	59.13	0.00	1.59	394.49	0.00	0.40	6.16	0.77	0.62	0.106
Chemicals	279	0.47	85.67	0.00	1.67	1212.45	0.00	0.70	6.59	0.95	0.93	0.027
Rubber and plastic	108	0.39	21.90	0.00	0.69	93.68	0.00	0.17	5.65	1.18	0.86	0.034
Other non-metallic products	95	0.19	53.87	0.00	1.38	16824.12	0.00	0.63	9.33	1.15	0.88	0.041
Metallic products	85	0.46	361.42	0.00	1.12	2496.12	0.00	0.39	8.07	1.03	0.93	0.622
Fabricated metal products	76	0.45	86.49	0.00	1.51	959.18	0.00	0.58	6.70	0.98	0.49	0.894
Machinery except electrical	78	0.46	183.44	0.00	1.21	988.14	0.00	0.23	7.41	0.83	0.95	0.112
Electric machinery	70	0.52	113.83	0.00	0.84	313.88	0.01	0.19	6.65	1.10	0.20	0.684
Professional and scientific equipment	15	1.06	5494.80	0.83	1.06	5494.80	0.83	0.00	9.56	0.77	0.83	0.954
Transport equipment	34	0.46	41.97	0.07	1.11	485.10	0.00	0.53	5.92	1.41	0.78	
other manufacturing	142	0.41	323.44	0.00	1.06	1977.84	0.00	0.18	8.24	0.91	0.81	0.081
Average		0.47	443.71		1.28	2395.75			7.16	1.01		

Notes. The values for x_{min} have been divided by 1000

Appendices

A. Closed Economy

Useful Formulas

$$\bar{r}_s = r(\tilde{\varphi}_s) = \sigma u_s f_s h_s^{\sigma_s - 1} \tag{A.1}$$

$$\bar{t}_s = t_s(\tilde{\varphi}_s) = \tau \left(u_s f_s h_s^{\sigma_s - 1} - \delta_s w f_s \right) \tag{A.2}$$

$$\frac{\partial u_s}{\partial \tau} = \frac{(1-\delta_s)}{(1-\tau)^2} \gtreqless 0 \tag{A.3}$$

$$\frac{\partial u_s}{\partial \delta_{s'}} = -\frac{\tau}{1-\tau} < 0 \qquad \text{if s=s', otherwise 0} \tag{A.4}$$

$$\frac{\partial \bar{r}_s}{\partial \delta_s} = \sigma_s f_s \left(h_s^{\sigma_s - 1} \frac{\partial u_s}{\partial \delta_s} + u_s \frac{\partial h_s^{\sigma_s - 1}}{\partial \delta_s} \right) \qquad \text{if s=s', otherwise 0} \tag{A.5}$$

$$\frac{\partial \bar{r}_s}{\partial \tau} = \sigma_s f_s \left(h_s^{\sigma_s - 1} \frac{\partial u_s}{\partial \tau} + u_s \frac{\partial h_s^{\sigma_s - 1}}{\partial \tau} \right) \tag{A.6}$$

$$\frac{\partial h_s^{\sigma_s - 1}}{\partial x} = (\sigma_s - 1) h_s^{\sigma_s - 1} \left[\frac{\partial \varphi_s^*}{\partial x} \frac{1}{\varphi_s^*} \left[\xi^s_{\tilde{\varphi}_s, \varphi_s^*} - 1 \right] \right] \tag{A.7}$$

$$\frac{\partial \tilde{\varphi}_s}{\partial \varphi_s^*} = \frac{z(\varphi_s^*)\tilde{\varphi}_s}{(\sigma - 1)(1 - Z_s(\varphi_s^*))} \left[1 - h_s^{1-\sigma} \right] \tag{A.8}$$

Elasticities As mentioned in the paper, let $\xi^s_{x,y}$ be the elasticity of variable x with respect to y for sector s.

$$\xi^s_{\tilde{\varphi}_s, \varphi^*} = \frac{z(\varphi_s^*)\varphi_s^*}{(\sigma - 1)(1 - Z(\varphi_s^*)} \left[1 - h_s^{1-\sigma} \right] \tag{A.9}$$

$$\xi^s_{M_s, \delta_{s'}} = \frac{\sum_{i=1}^S \frac{\partial T_i}{\partial \delta_{s'}} \delta_{s'}}{\left(wL + \sum_{i=1}^S T_i - q_0^G \right)} - \left[\frac{-\tau \delta_s}{(1 - \delta_s \tau)} + (\sigma - 1) \left(\xi_{\varphi_s^*, \delta_{s'}} \left[\xi_{\tilde{\varphi}_s, \varphi_s^*} - 1 \right] \right) \right] \tag{A.10}$$

$$\xi^s_{M_s, \delta_{s'}} = \frac{\sum_{i=1}^S \frac{\partial T_i}{\partial \delta_{s'}} \delta_{s'}}{\left(wL + \sum_{i=1}^S T_i - q_0^G \right)} \qquad \text{if s} \neq \text{s'} \tag{A.11}$$

$$\xi^s_{M_s, \tau} = \frac{\sum_{i=1}^S \frac{\partial T_i}{\partial \tau} \tau}{\left(wL + \sum_{i=1}^S T_i - q_0^G \right)} - \left[\frac{(1-\delta_s)\tau}{(1-\tau)(1 - \delta_s \tau)} + (\sigma - 1) \left(\xi_{\varphi^*, \tau} \left[\xi_{\tilde{\varphi}_s, \varphi_s^*} - 1 \right] \right) \right] \tag{A.12}$$

A.1. Optimal Taxes in the Closed Model

The FOCs for δ_i and τ are rewritten into:

$$\alpha_i\left[\frac{\tau\delta_i}{(1-\delta_i\tau)(1-\sigma_i)}-\xi_{\varphi_i^*,\delta_i}\right]=\tilde{\lambda}M_i\tau\delta_i f_i\left[\frac{-w}{1-\delta_i\tau}+(\sigma_i-1)\xi_{\varphi_i^*,\delta_i}(\xi_{\tilde{\varphi}_i,\varphi_i^*}-1)w\right] \tag{A.13}$$

$$\begin{aligned}&\sum_{i=1}^{S}\alpha_i\left(\frac{-(1-\delta_i)\tau}{(1-\tau)(1-\delta_i\tau)(1-\sigma_i)}-\xi_{\varphi_{s'}^*,\tau}\right)=\\ &\tilde{\lambda}\sum_{i=1}^{S}\left[M_i\tau w f_i\left((\sigma_i-1)\xi_{\varphi_i^*,\tau}(\xi_{\tilde{\varphi}_i,\varphi_i^*}-1)\delta_i+u_ih_i^{\sigma_i-1}-\delta_i\left(\frac{1-2\tau+\delta_i\tau^2}{(1-\tau)(1-\delta_i\tau)}\right)\right)\right]\end{aligned} \tag{A.14}$$

Pareto Distribution

Assuming productivities follow a Pareto distribution, i.e:

$$Z_i(\varphi)=1-\left(\frac{\varphi_{min,i}}{\varphi}\right)^{k_i}$$

Under this distribution, the variables needed to solve have the following expressions:

$$\tilde{\varphi}_i=\left(\frac{k_i}{k_i-(\sigma_i-1)}\right)^{\frac{1}{\sigma_i-1}}\varphi_i^* \tag{A.15}$$

$$\varphi_i^*=\left[\left(\frac{\sigma_i-1}{k_i-(\sigma_i-1)}\right)\left(\frac{f_i(1-\delta_i\tau)}{\psi f_{e,i}}\right)\right]^{1/k_i}\varphi_{min,i} \tag{A.16}$$

$$\xi_{\varphi_i^*,\delta_i}=\frac{-\tau\delta_i}{k_i(1-\delta_i\tau)}=\xi_{\varphi_i^*,\tau} \tag{A.17}$$

Plugging these values into equation A.13:

$$1-\delta_i\tau=\tilde{\lambda}(1-\tau)\rho_i wL \tag{A.18}$$

Such relation is used to find the optimal statutory corporate tax rate through equation A.14:

$$1-\tau=\left[\sum_{i=1}^{S}\frac{\alpha_i}{k_i}\right]\left[\tilde{\lambda}wL\sum_{i=1}^{S}\frac{\alpha_i\rho_i}{k_i}\right]^{-1} \tag{A.19}$$

Log-normal Distribution

Under this distribution, the variables needed to solve the model must be found through numerical methods. To solve for $\tilde{\varphi}_i$ define:

$$d_i = \frac{(log(\varphi_i^*) - m_i)}{v_i} \tag{A.20}$$

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}x^2) \tag{A.21}$$

where m_i, v_i are the parameters for the log-normal distribution of productivities for sector i. The function $\Phi(x)$ is the CDF for the standard normal distribution. Using, these variables:

$$\tilde{\varphi}_i^{\sigma_i - 1} = \frac{1}{1 - Z_i(\varphi_i^*)} \int_{\varphi_i^*}^{\infty} \varphi^{\sigma_i - 1} z(\varphi) d\varphi \tag{A.22}$$

$$= \exp\left(m_i(\sigma_i - 1) + \frac{((\sigma_i - 1)v_i)^2}{2} \right) \frac{\Phi((\sigma_i - 1)v_i - d_i)}{\Phi(-d_i)} \tag{A.23}$$

$$= A_i g(\varphi_i^*) \tag{A.24}$$

Equation A.23 is obtained through various substitutions in the integral, as well as using the symmetry of the normal distribution.[28] The productivity cutoff φ_s^* is found by solving:

$$\frac{A_i g_i(\varphi_i^*)}{(\varphi_i^*)^{\sigma - 1}} = \frac{\psi f_{e,i}}{(1 - \delta_i \tau)\Phi(-d_i) f_i} + 1 \tag{A.25}$$

In order to solve for the optimal rates we must find a formula for $\xi_{\varphi_i^*, \delta_i}$. This is accomplish by using A.7,A.9 and the ZP and FE conditions.

$$\xi_{\varphi_i^*, \delta_i} = \frac{\psi f_{e,i}}{X_i(1 - \sigma_i)} \left(\frac{\tau \delta_i}{1 - \tau \delta_i} \right) = \xi_{\varphi_i^*, \tau} \tag{A.26}$$

$$X_i = \psi f_{e,i} + (1 - \delta_i \tau)\Phi(-d_i) f_i \tag{A.27}$$

Using the above formula, equations A.13 result in the following relationship:

$$\frac{1}{(1 - \tau)\rho_i \lambda w L} = \frac{\psi f_{e,i} + \Phi(-d_i) f_i}{X_i} - \frac{\psi f_{e,i} \phi(-d_i)}{X_i \Phi(-d_i) v_i} \xi_{\varphi_i^*, \delta_i} \tag{A.28}$$

[28]The step by step derivation can be provided upon request

while equation A.14 can be simplified to:

$$\sum_{i=1}^{S}\frac{\alpha_i}{\sigma_i-1}\left(\frac{\tau}{(1-\tau)X_i}\right)(\psi f_{e,i}+(1-\delta_i)\Phi(-d_i)f_i)$$
$$=\tilde{\lambda}\tau\sum_{i=1}^{S}M_i w f_i\left[\delta_i\left(-\frac{(\psi f_{e,i}+\Phi(-d_i)f_i)}{X_i}+\frac{\tau}{1-\tau}+\frac{\psi f_{e,i}\phi(-d_i)}{X_i\Phi(-d_i)v_i}\xi_{\varphi_i^*,\delta_i}\right)+u_i h_i^{\sigma-1}\right]$$

which simplifies to:

$$1-\tau=\left[\sum_{i=1}^{S}\frac{\alpha_i}{\sigma_i-1}\right]\left[\tilde{\lambda}wL\sum_{i=1}^{S}\frac{\alpha_i}{\sigma_i}\left(\frac{\psi f_{e,i}+\Phi(-d_i)f_i}{X_i}\right)\right]^{-1} \tag{A.29}$$

Thus the solution to the problem is found by solving the system of $S+1$ equations given by A.28 and A.29.

B. Proofs

B.1. Proof of Proposition 1

For any non-degenerate distribution the mean of the random variable is greater than the minimum value of the support. Thus $\tilde{\varphi}>\varphi^*$ which implies $h>1 \implies h^{-1}<1$. Raising both sides of the inequality by the positive number $\sigma-1$ is use to show that $1-h^{1-\sigma}$ is greater than zero. Thus equation A.9 consist of positive factors and hence greater than zero.

For the second part, assume that productivities follow a Pareto distribution with $x_{min,s}=\varphi_{min,s}$ and shape parameter k_s.Then

$$\tilde{\varphi}_s=\left[\frac{k_s}{k_s-(\sigma_s-1)}\right]^{\frac{1}{\sigma_s-1}}\varphi_s^*$$
$$\frac{\partial\tilde{\varphi}_s}{\partial\varphi_s^*}=\left[\frac{k_s}{k_s-(\sigma_s-1)}\right]^{\frac{1}{\sigma_s-1}}$$

Using the above equations it is clear that $\xi_{\tilde{\varphi},\varphi^*}$ is exactly one.

B.2. Proof of Proposition 2

Assume the government budget constraint is binding and therefore the number of firms in equilibrium is: $M_s=\dfrac{wL}{\sigma_s u_s f_s h_s^{\sigma_s-1}}$. Let $s\neq s'$, then the binding budget assumption implies that equation A.11 is equal to zero for any distribution of productivities.

Now assume that $s = s'$ for some $s' \in S$. For a any productivity distribution, equation A.10 simplifies to:

$$\xi_{M_s,\delta_{s'}} = -\left[\frac{-\tau\delta_s}{(1-\delta_s\tau)} + (\sigma_s - 1)\left(\xi_{\varphi_s^*,\delta_{s'}}\left[\xi_{\tilde{\varphi}_s,\varphi_s^*} - 1\right]\right)\right]$$

Proposition 1 says that $\xi^P_{\tilde{\varphi},\varphi^*} \equiv 1$, therefore:

$$\xi_{M_s,\delta_{s'}} - \xi^P_{M_s,\delta_{s'}} = -(\sigma_s - 1)\left(\xi_{\varphi_s^*,\delta_{s'}}\left[\xi_{\tilde{\varphi}_s,\varphi_s^*} - 1\right]\right)$$

The term $(\sigma - 1)\xi_{\varphi^*_{s'},\delta_{s'}}$ is less than zero since the productivity cutoff is negatively related to the depreciation rate for its sector. Using the appropriate assumptions on $\xi_{\tilde{\varphi}_s,\varphi_s^*}$ gives the inequalities between both elasticities.

It remains to show that the elasticity spawn from a Pareto distribution is greater than zero. The formula for such elasticity is:

$$\xi^P_{M_{s'},\delta_{s'}} = \frac{\tau\delta_{s'}}{1-\delta_{s'}\tau}$$

by assumption, $\delta_s\tau < 1$ for all sectors, and hence $\xi^P_{M_s,\delta_s}$ is positive.

B.3. Proof of Proposition 3

Only the first bullet point is proved as the second one follows a similar argument. Under a binding government constraint, equation A.12 simplifies to:

$$\begin{aligned}
\xi_{M_s,\tau} &= -\frac{(1-\delta_s)\tau}{(1-\tau)(1-\delta_s\tau)} - (\sigma_s - 1)\left(\xi_{\varphi^*,\tau}\left[\xi_{\tilde{\varphi}_s,\varphi_s^*} - 1\right]\right) \\
\xi^P_{M_s,\tau} &= -\frac{(1-\delta_s)\tau}{(1-\tau)(1-\delta_s\tau)}
\end{aligned}$$

If $\delta_s \leq 1$, then clearly $\xi^P_{M_s,\tau} \leq 0$, with strict inequality if $\delta_s < 1$. Since $\xi_{\varphi_{s'},\delta_{s'}} = \xi_{\varphi_{s'},\tau}$ (this is shown in the next proof), I use a similar argument for the proof of proposition 2 to establish the inequalities between ξ_M and ξ^P_M. Assuming $\xi_{\tilde{\varphi},\varphi^*} < 1$ and proposition 2, the following equality is obtained:

$$\xi_{M_s,\tau} < \xi^P_{M_s,\tau} \leq 0$$

On the other hand, if $\xi_{\tilde{\varphi},\varphi^*} < 1$ then $\xi_{M_s,\tau} > \xi^P_{M_s,\tau}$; and therefore the sign of the elasticity of firms to taxes under a distribution that is not Pareto is indeterminate. The exception being $\delta = 1$, which then implies such elasticity to be positive since $\xi^P_{M,\tau} = 0$

B.4. Proof of Proposition 4

The first step is to show the following equality between elasticities

Claim: $\xi_{\varphi_i^*,\delta_i} = \xi_{\varphi_i^*,\tau}$

Proof. The ZP and FE conditions imply that the equilibrium φ_s^* must solve the equation:

$$h_s^{\sigma-1} = \frac{\psi F_{e,s}}{(1 - Z_s(\varphi_s^*))(1-\delta_s\tau)f_s} + 1$$

Take the derivative with respect to τ as well as δ_s. The ratio of such derivatives is:

$$\frac{\frac{\partial h_s^{\sigma-1}}{\partial \tau}}{\frac{\partial h_s^{\sigma-1}}{\partial \delta_s}} = \frac{z_s(\varphi_s^*)\frac{\partial \varphi^*}{\partial \tau}(1-\delta_s\tau) + (1 - Z_s(\varphi_s^*))\delta_s}{z_s(\varphi_s^*)\frac{\partial \varphi^*}{\partial \delta_s}(1-\delta_s\tau) + (1 - Z_s(\varphi_s^*))\tau}$$

By equation A.7:

$$\frac{\frac{\partial h_s^{\sigma-1}}{\partial \tau}}{\frac{\partial h_s^{\sigma-1}}{\partial \delta_s}} = \left(\frac{\partial \varphi_s^*}{\partial \tau}\right)\left(\frac{\partial \varphi_s^*}{\partial \delta_s}\right)^{-1}$$

Set the last two equation equal to each other and rearrange to obtain:

$$\tau\left(\frac{\partial \varphi_s^*}{\partial \tau}\right) = \delta_s\left(\frac{\partial \varphi_s^*}{\partial \delta_s}\right)$$

$$\xi_{\varphi_s^*,\tau} = \xi_{\varphi_s^*,\delta_s}$$

■

After proving the above claim, the FOCs (eq. 3.1 and 3.2) are re-written into:

$$\alpha_{s'}\left(\frac{\tau\delta_{s'}}{(1-\delta_{s'}\tau)(1-\sigma_i)} - \xi_{\varphi_{s'}^*,\delta_{s'}}\right) = \tilde{\lambda}M_{s'}\left(\xi_{M_{s'},\delta_{s'}}\bar{t}_{s'} + \frac{\partial \bar{t}_{s'}}{\partial \delta_{s'}}\delta_{s'}\right) \tag{B.1}$$

$$\sum_{i=1}^{S}\alpha_i\left(\frac{-(1-\delta_i)\tau}{(1-\tau)(1-\delta_i\tau)(1-\sigma_i)} - \xi_{\varphi_{s'}^*,\tau}\right) = \tilde{\lambda}\left[\sum_{i=1}^{S}M_i\left(\xi_{M_i,\tau}\bar{t}_i + \frac{\partial \bar{t}_{i'}}{\partial \tau}\tau\right)\right] \tag{B.2}$$

Adding equation B.1 across all sectors and using the equality of the claim results in:

$$\sum_{i=1}^{S}\alpha_i\left(\frac{\tau(1-\delta_i\tau)}{(1-\delta_i\tau)(1-\tau)(1-\sigma_i)}\right) = \tilde{\lambda}\sum_{i=1}^{S}M_i\left[\left(\xi_{M_i,\delta_i} - \xi_{M_i,\tau}\right)\bar{t}_i + \left(\frac{\partial \bar{t}_i}{\partial \delta_i}\delta_i - \frac{\partial \bar{t}_i}{\partial \tau}\tau\right)\right]$$

$$\sum_{i=1}^{S}\frac{\alpha_i\tau}{(1-\tau)(1-\sigma_i)} = \tilde{\lambda}\sum_{i=1}^{S}M_i\left[\left(\frac{\tau}{1-\tau}\bar{t}_i\right) + \left(\frac{\partial \bar{t}_i}{\partial \delta_i}\delta_i - \frac{\partial \bar{t}_i}{\partial \tau}\tau\right)\right] \tag{B.3}$$

Next, the remainder derivatives are computed:

$$
\begin{aligned}
\frac{\partial \bar{t}_i}{\partial \delta_i}\delta_i &= \tau \delta_i w f_i \left(\frac{\partial u_i}{\partial \delta_i} h_i^{\sigma_i - 1} + \frac{\partial h_i^{\sigma_i-1}}{\partial \delta_i} u_i - 1 \right) \\
\frac{\partial \bar{t}_i}{\partial \tau}\tau &= \tau w f_i \left[\left(\frac{\partial u_i}{\partial \tau} h_i^{\sigma_i - 1} + \frac{\partial h_i^{\sigma_i-1}}{\partial \tau} u_i \right) \tau + u_i h_i^{\sigma_i-1} - \delta_i \right] \\
\frac{\partial \bar{t}_i}{\partial \delta_i}\delta_i - \frac{\partial \bar{t}_i}{\partial \tau}\tau &= \tau w f_i \left[h_i^{\sigma_i-1} \left(\frac{\partial u_i}{\partial \delta_i}\delta_i - \frac{u_i}{\tau}\tau \right) + u_i \left(\frac{\partial h_i^{\sigma_i-1}}{\partial \delta_i}\delta_i - \frac{\partial h_i^{\sigma_i-1}}{\partial \tau}\tau \right) - u_i h_i^{\sigma_i-1} \right] \\
&= \tau w f_i \left[h_i^{\sigma-1} u_i \left(\frac{-\tau}{1-\tau} \right) + 0 - u_i h_i^{\sigma_i-1} \right] \\
&= \tau w f_i \left(h_i^{\sigma_i-1} u_i \frac{-1}{1-\tau} \right)
\end{aligned}
$$

Replacing terms in equation B.3 gives the formula for λ

$$
\sum_{i=1}^{S} \frac{\alpha_i}{\sigma_i - 1} = \tilde{\lambda} \left[\sum_{i=1}^{S} -M_i \bar{t}_i + \frac{\alpha_i (wL)}{\sigma_i} \right]
$$

$$
\tilde{\lambda} = \frac{\sum_{i=1}^{S} \frac{\alpha_i}{\sigma_i - 1}}{wL \sum_{i=1}^{S} \frac{\alpha_i}{\sigma_i} - p_0^G q_0^G} \tag{B.4}
$$

B.5. Proof of Proposition 6

Let $\alpha_i = \alpha$, $k_i = k \; \forall i$ and productivities are Pareto distributed. Equation A.19 simplifies to :

$$
1 - \tau = S \left[\tilde{\lambda} w L \sum_{i}^{S} \rho_i \right]^{-1}
$$

Use the above to simplify the equation for $\delta_{i'}$ and add across all sectors:

$$
\begin{aligned}
\sum_{i'}^{S} (1 - \delta_{i'} \tau) &= \sum_{i'}^{S} \left(\rho_{i'} S \sum_{i}^{S} \rho_i \right) \\
S - \tau \sum_{i'=1}^{S} \delta_{i'} &= S
\end{aligned}
$$

Since $G > 0 \implies \tau > 0$ hence $0/\tau = 0 \implies \sum \delta_i = 0$ ■

B.6. Proof of Proposition 7

(i) Pareto Economy: Assume $k_i = \bar{k}, \sigma_i = \bar{\sigma} \quad i \in S$, then $1-\tau = \left(\tilde{\lambda} w L \bar{\rho}\right)^{-1}$. From the optimality equation for δ:

$$\delta_i = \frac{1 - \tilde{\lambda}\bar{\rho} w L(1-\tau)}{\tau} = \frac{0}{\tau} = 0 \qquad \forall i$$

The equation above is valid since $\tau > 0$.

(ii) Log-normal Economy: Assume sectors are completely symmetric, hence no sector subscript will be needed for the model parameters. Equation A.29 implies:

$$1 - \tau = \frac{1}{\rho\tilde{\lambda} w L A}$$
$$A = \frac{\psi F_e + \Phi(-d) f}{X}$$

Replacing $(1-\tau)$ in equation A.28, leads to:

$$\frac{1}{A} = A - \frac{\psi F_e \phi(-d)}{X\Phi(-d)v}\xi_{\tilde{\varphi}^*,\delta} = A - B$$

There are 3 possible case for δ, with each determining is A if above, below, or equal to 1. We show that cases of $\delta \neq 0$ produce a contradiction.

Case 1: Assume $\delta > 0$. This implies $A > 1$ and $1/A < 1$. Using the formula for the elasticity, we can see that $B < 0$. Hence, the equality can't hold as the LHS is less than one, while the RHS is greater than 1.

Case 2: Assume $\delta < 0$. Just as the above case, the equality can't hold since $A < 1, 1/A > 1$ and $B > 0$.

Case 3: Assume $\delta = 0$. In this case, $A = 1 \implies 1/A = 1$. Since $\delta = 0$, the elasticity $\xi_{\tilde{\varphi}^*,\delta}$ is equal to 0. Hence, the equality holds as $1 = 1$. Therefore, the only solution to the optimal tax rate problem is $\delta = 0$ for all sectors.

C. Equilibrium: Open Economy with Asymmetric Countries

The world consists of N countries whose households have the same utility function form but its parameters (σ, α) can vary across countries. Firms can export their products by paying an iceberg trade cost (θ_s^{ij}) and an additional export fixed cost $(f_{ex,s}^{ij})$. The subscript of the variables denote the sector while the upper script denote the flow of the good between countries: j is the source country while i is the destination. This notation will be used for other variable as well when the need to specify the flow between countries arises. Companies in j that want to export to country i have to pay a fixed cost $f^{ij}_{ex,s}$.

Since the elasticity of substitution can be heterogeneous across countries, it implies that the markup charged by firms is different in each country leading to the pricing decision rule:

$$p^{ij}_s(\varphi) = \theta^{ij}_s \frac{w}{\rho^i_s \varphi}$$

where the wage (w) is the same across countries.[29]

Let $\pi^j_{d,s}(\varphi)$ represent the domestic profit of firms in j and $\pi^{ij}_{ex,s}(\varphi)$ represents the profits from exporting into i.

$$\pi^j_{d,s}(\varphi) = (1-\tau^j)\left(\frac{r^j_{d,s}(\varphi)}{\sigma^j_s} - u^j_s w f^j_s\right)$$
$$\pi^{ij}_{d,s}(\varphi) = (1-\tau^j)\left(\frac{r^{ij}_{ex,s}(\varphi)}{\sigma^i_s} - u^j_s w f^{ij}_{ex,s}\right)$$

C.1. Equiibrium and Aggregation

Let $\varphi^j_{d,s}$ be the cutoff productivity to enter the domestic market while $\varphi^{ij}_{ex,s}$ is the cutoff productivity of the marginal firm that decides to export to country i. Unlike the usual symmetric country versions, there is a set of export productivity cutoffs for each country. Moreover, if a firm enters the export market for an arbitrary country, it does not necessarily imply that it will serve all other export markets. Using $\tilde{\varphi}()$ (equation 2.7) we can define the average productivity of all firms producing and selling in j as $\tilde{\varphi}^j_d = \tilde{\varphi}^j(\varphi^j_d)$ and, the productivity of the firms exporting by $\tilde{\varphi}^{ij}_{ex} = \tilde{\varphi}^i(\varphi^{ij}_{ex})$

Let $i \neq j$, the number of firms (in sector s) producing in country j is M^j_s and the number of firms choosing to export to country i is $M^{ij}_{ex,s}$. The total number of varieties, in sector s, available to consumers in country j is $M^j_{tot} = M^j + \sum_{i \neq j} M^{ji}_{ex}$.

[29]This is justified by using a homogeneous good that is freely traded and use such good as the numéraire.

The average total productivity in j ($\tilde{\varphi}^j_{tot}$) and the price index is :

$$\tilde{\varphi}^j_s = \left[\frac{1}{M^j_{tot,s}}\left(M^j_s\left(\tilde{\varphi}^j_s\right)^{\sigma^j_s-1} + \sum_{i\neq j}\left(\left(\theta^{ji}_s\right)^{-1}\tilde{\varphi}^{ji}_{ex,s}\right)^{\sigma^j_s-1}\right)\right]^{\frac{1}{\sigma^j_s-1}}$$

$$\mathbb{P}^j_s = \left[\frac{1}{1-Z^j_s(\varphi^j_{d,s})}\int_{\varphi^j_{d,s}}^{\infty} p_s(\varphi)^{1-\sigma^j_s} M^j_s z^j_s(\varphi) + \sum_{i\neq j}\frac{1}{1-Z^i_s(\varphi^{ji}_{ex,s})}\int_{\varphi^{ji}_{ex,s}}^{\infty} p^{ji}_{ex,s}(\varphi)^{1-\sigma^j_s} M^{ji}_{ex,s} z^i_s(\varphi)\right]^{\frac{1}{1-\sigma^j_s}}$$

$$\mathbb{P}^j_s = \left(M^j_{tot,s}\right)^{\frac{1}{1-\sigma^j_s}} p_s(\tilde{\varphi}^j_{tot,s})$$

With those, aggregate and average functions for firm revenues and profits are:

$$R^j_s = M^j_s r^j_{d,s}(\tilde{\varphi}^j_{d,s}) + \sum_{i\neq j} M^{ij}_{ex,s} r^{ij}_{ex,s}(\tilde{\varphi}^{ij}_{ex,s})$$

$$\Pi^j_s = M^j_s \pi^j_{d,s}(\tilde{\varphi}^j_{d,s}) + \sum_{i\neq j} M^{ij}_{ex,s} \pi^{ij}_{ex,s}(\tilde{\varphi}^{ij}_{ex,s})$$

$$\bar{r}^j_s = r^j_{d,s}(\tilde{\varphi}^j_{d,s}) + \sum_{i\neq j} \kappa^{ij}_{ex,s} r^{ij}_{ex,s}(\tilde{\varphi}^{ij}_{ex,s})$$

$$\bar{\pi}^j_s = \pi^j_{d,s}(\tilde{\varphi}^j_{d,s}) + \sum_{i\neq j} \kappa^{ij}_{ex,s} \pi^{ij}_{ex,s}(\tilde{\varphi}^{ij}_{ex,s})$$

in which $\kappa^{ij}_{ex} = \dfrac{1-Z^j_s(\varphi^{ij}_{ex,s})}{1-Z^j_s(\varphi^j_{d,s})}$ is the conditional probability of a firm drawing a productivity that allows them to serve market i from country j. Also, $\kappa^{ij}_{ex} M^j_s = M^{ij}_{ex,s}$.

The above formulas are used to find the average profit as a function of $\varphi^j_{d,s}$ (productivity that generates zero profit from domestic operations) and $\varphi^{ij}_{ex,s}$ (productivity that generates zero profit from exporting to i).

$$\bar{\pi}^j_s = (1-\delta^j_s\tau^j)w\left[f^j_s\left(\left(\frac{\tilde{\varphi}^j_{d,s}}{\varphi^j_{d,s}}\right)^{\sigma^j_s-1} - 1\right) + \sum_{i\neq j}\kappa^{ij}_{ex} f^{ij}_{ex,s}\left(\left(\frac{\tilde{\varphi}^{ij}_{ex,s}}{\varphi^{ij}_{ex}}\right)^{\sigma^i_s-1} - 1\right)\right] \tag{C.1}$$

To solve for $\varphi^j_{d,s}$, the export cutoffs must be expressed as functions of such variable:

$$\varphi^{ij}_{ex,s} = \left[\left(\frac{\sigma^i_s f^{ij}_{ex,s}}{\sigma^j_s f^j_s}\right)\frac{Y^j_s}{Y^i_s}\frac{M^i_{tot,s}}{M^j_{tot,s}}\right]^{\frac{1}{\sigma^i_s-1}}\left(\frac{\varphi^j_{d,s}}{\tilde{\varphi}^j_{tot,s}}\right)^{\frac{\sigma^j_s-1}{\sigma^i_s-1}}\tilde{\varphi}^i_{tot,s}\theta^{ij}_s \tag{C.2}$$

where $Y_s = \alpha_s(wL)$ is the income spend in sector s. Plugging this formula into equation C.1 gives rise to zero profit condition for the open economy asymmetric model. The fixed entry (equation FE) remains the same. The export cutoff formula depends on the total number of firms in the destination country as well as the country where the firms is located.

The number of firms for sector s in country j is:

$$M_s^j = \frac{\alpha_s^j (wL^j + \sum_{s=1}^{S} \Pi_s^{\tau,j})}{\sigma_s^j \left(\frac{\bar{\pi}_s^j}{1-\tau^j} + u_s^j f_s^j \right) + w u_s^j \sum_{i \neq j} \kappa_{ex,s}^{ij} f_{ex,s}^{ij} \left(\sigma_s^j + (\sigma_s^i - \sigma_s^j) \frac{\tilde{\varphi}_{ex,s}^{ij}}{\varphi_{ex,s}^{ij}} \right)} \tag{C.3}$$

Thus, for each sector, in each country, we solve 2 equations ZPC = FE and C.3 with N auxiliary equations (C.2). This leads to a system of $N \times S \times (N+2)$ equations that are solved simultaneously to give rise to the equilibrium of the model. For the case of Pareto distributed productivities, the system of equations reduces to $N \times S \times 2$ since the ratio $\tilde{\varphi}_{ex}/\varphi_{ex}$ is constant.

D. Open Economy Symmetric Countries: proofs

Before giving the proofs of each particular distribution, I start with some derivations for the general case. The model uses only two countries j, i but results extend to N-countries.

First some notation simplification. Since countries are symmetric, the upper(sub) scripts identify the country are not needed anymore. Thus, $\varphi_{d,s}^*, \varphi_{ex,s}^*$ denote the productivity cutoff to enter the domestics market and to export respectively. Fixed costs of production ($f_{d,s}$), export ($f_{ex,s}$), and entry $F_{e,s}$. To simplify notation further, $\Upsilon_{d,s} = \Upsilon_s(\varphi_{d,s}^*)$ and $\Upsilon_{ex,s} = \Upsilon_s(\varphi_{ex,s}^*)$.[30] The subscript specifying the sector (s) will be used only when necessary.

Under the symmetric country assumption, the export cutoff is simply:

$$\varphi_{ex}^* = \left(\frac{f_{ex}}{f_d} \right)^{\frac{1}{\sigma-1}} \theta \varphi_d^* \tag{D.1}$$

which implies that $\xi_{\varphi_{ex}^*,x} = \xi_{\varphi_d^*,x}$ for $x = \tau, \delta$.

The following auxiliary variables make the derivations a little cleaner:

$$\tilde{f} = f_d + \kappa^{ex} f_{ex}$$

$$\widetilde{fh} = f_d h_d^{\sigma-1} + \kappa^{ex} f_{ex} h_{ex}^{\sigma-1}$$

$$\tilde{X} = \psi F_e + \tilde{f}(1 - Z(\varphi_d^*))(1 - \delta\tau)$$

[30] As a reminder:

$$\Upsilon_{js}(x) = \frac{z_{js}(x)}{1 - Z_{js}(x)} x$$

where Z, z represent the CDF and PDF of the random distribution.

notice that the ZP = FE equation equals:

$$\widetilde{fh} = \frac{\tilde{X}}{(1 - Z(\varphi_d^*))(1 - \delta\tau)}$$

The derivatives of $\widetilde{fh}$ comes often so its expression, for $x = \tau, \delta$, is given below along with other derivatives that appear often in the simplifications :

$$\frac{\partial \widetilde{fh}}{\partial x} x = \frac{\psi F_e}{(1 - Z(\varphi_d^*))(1 - \delta\tau)} \left[\Upsilon_d \xi_{\varphi_d^*, x} + \frac{\delta\tau}{1 - \delta\tau} \right] + f_{ex} \frac{\partial \kappa^{ex}}{\partial x} x \tag{D.2}$$

$$\frac{\partial \kappa^x}{\partial x} x = \kappa^x \xi_{\varphi_d^*, x} \left(\Upsilon_d - \Upsilon_{ex} \right) \tag{D.3}$$

$$\frac{\partial \tilde{\varphi}_d^{\sigma-1}}{\partial x} x = \tilde{\varphi}_d^{\sigma-1} \xi_{\varphi_d^*, x} \Upsilon_d \left[1 - h_d^{1-\sigma} \right] \tag{D.4}$$

$$\frac{\partial \tilde{\varphi}_{ex}^{\sigma-1}}{\partial x} x = \tilde{\varphi}_{ex}^{\sigma-1} \xi_{\varphi_{ex}^*, x} \Upsilon_{ex} \left[1 - h_{ex}^{1-\sigma} \right] \tag{D.5}$$

With the above, the elasticity of the number of firms (eq. 5.3 and 5.4) becomes:

$$\begin{aligned} \xi_{M,\delta} &= \frac{\tau\delta}{1 - \tau\delta} - \frac{1}{\widetilde{fh}} \frac{\partial \widetilde{fh}}{\partial \delta} \delta \\ &= \tau\delta \left(\frac{\tilde{f}(1 - Z_d)}{\tilde{X}} \right) - \frac{\psi F_e}{\tilde{X}} \left[\Upsilon_d \xi_{\varphi_d^*, \delta} \right] - \frac{f_{ex}}{\widetilde{fh}} \frac{\partial \kappa^{ex}}{\partial \delta} \delta \end{aligned} \tag{D.6}$$

$$\begin{aligned} \xi_{M,\tau} &= \frac{-\tau(1 - \delta)}{(1 - \tau\delta)(1 - \tau)} - \frac{1}{\widetilde{fh}} \frac{\partial \widetilde{fh}}{\partial \tau} \tau \\ &= \frac{-\tau}{(1 - \tau)\tilde{X}} \left(\psi F_e + \tilde{f}(1 - Z_d)(1 - \delta) \right) - \frac{\psi F_e}{\tilde{X}} \left[\Upsilon_d \xi_{\varphi_d^*, \tau} \right] - \frac{f_{ex}}{\widetilde{fh}} \frac{\partial \kappa^{ex}}{\partial \tau} \tau \end{aligned} \tag{D.7}$$

Furthermore, the elasticities $\xi_{\tilde{\varphi}^*, x}$ are easily found. Start with the derivative of $\widetilde{fh}$:

$$\begin{aligned} \frac{\partial \widetilde{fh}}{\partial x} x &= f_d \frac{\partial h_d^{\sigma-1}}{\partial x} x + f_{ex} \left(\kappa^{ex} \frac{\partial h_{ex}^{\sigma-1}}{\partial x} x + (h_{ex}^{\sigma-1} - 1) \frac{\partial \kappa_{ex}}{\partial x} x \right) \\ &= \xi_{\varphi_d^*, x} f_d \left(\Upsilon_d (h_d^{\sigma-1} - 1) - \frac{(\sigma - 1)}{h_d^{1-\sigma}} \right) + \xi_{\varphi_{ex}^*, x} f_{ex} \kappa^{ex} \left(\Upsilon_{ex} (h_{ex}^{\sigma-1} - 1) - \frac{(\sigma - 1)}{h_{ex}^{1-\sigma}} + \left(\Upsilon_d - \Upsilon_{ex} \right) h_{ex}^{\sigma-1} \right) \\ &= \xi_{\varphi_d^*, x} \left[\Upsilon_d \widetilde{fh} - \Upsilon_d f_d - (\sigma - 1) \widetilde{fh} - \Upsilon_{ex} f_{ex} \kappa^{ex} \right] \end{aligned} \tag{D.8}$$

Equating the above expression with its counterpart of equation D.2, results in:

$$\xi_{\varphi_d^*,x}\Upsilon_d\left[\widetilde{fh}-\frac{\psi F_e}{(1-Z_d)(1-\delta\tau)}\right]=\xi_{\varphi_d^*,x}\Upsilon_d\tilde{f}+\frac{\psi F_e}{(1-Z_d)(1-\delta\tau)}\frac{\delta\tau}{1-\delta\tau}+\xi_{\varphi_d^*,x}\frac{(\sigma-1)\tilde{X}}{(1-Z_d)(1-\delta\tau)}$$

$$-\frac{\psi F_e}{1-Z_d}\frac{\delta\tau}{1-\delta\tau}=\frac{(\sigma-1)\tilde{X}}{(1-Z_d)}\xi_{\varphi_d^*,x}$$

$$\frac{\psi F_e}{(1-\sigma)\tilde{X}}\frac{\delta\tau}{1-\delta\tau}=\xi_{\varphi_d^*,x} \tag{D.9}$$

Simplify the FOCs of the general model. As a remainder, the constant a_{js} becomes:

$$a_{js}=\tilde{\varphi}_{jjs}^{\sigma_s-1}+\kappa_{ijs}^{ex}\frac{M_{is}}{M_{js}}\left(\hat{\theta}_{ijs}^{-1}\tilde{\varphi}_{ijs}\right)^{\sigma_s-1}=\tilde{\varphi}_{d,s}^{\sigma_s-1}+\kappa_s^{ex}\left(\theta^{-1}\tilde{\varphi}_{ex,s}\right)^{\sigma_s-1} \tag{D.10}$$

Start with the LHS of equation 5.1 (the FOC for δ):

$$\left(\frac{\alpha a^{-1}}{\sigma-1}\right)\left(\frac{\xi_{M,\delta}}{\tilde{\varphi}_d^{1-\sigma}}+\frac{\partial\tilde{\varphi}_d^{\sigma-1}}{\partial\delta}\delta+\frac{M_i}{M_j}\hat{\theta}^{1-\sigma}\left(\frac{\partial\kappa^x}{\partial\delta}\delta\tilde{\varphi}_{ex}^{\sigma-1}+\kappa^{ex}\left(\frac{\xi_{M_i,\delta}}{\tilde{\varphi}_{ex}^{1-\sigma}}+\frac{\partial\tilde{\varphi}_{ex}^{\sigma-1}}{\partial\delta}\delta\right)\right)\right)$$

$$\left(\frac{\alpha a^{-1}}{\sigma-1}\right)\left(\frac{\xi_{M,\delta}}{\tilde{\varphi}_d^{1-\sigma}}+\frac{\partial\tilde{\varphi}_d^{\sigma-1}}{\partial\delta}\delta+\theta^{1-\sigma}\left(\frac{\partial\kappa^{ex}}{\partial\delta}\delta\tilde{\varphi}_{ex}^{\sigma-1}+\kappa^{ex}\left(\frac{\xi_{M,\delta}}{\tilde{\varphi}_{ex}^{1-\sigma}}+\frac{\partial\tilde{\varphi}_{ex}^{\sigma-1}}{\partial\delta}\delta\right)\right)\right)$$

$$\left(\frac{\alpha a^{-1}}{\sigma-1}\right)\left(\xi_{M,\delta}a+\frac{\partial\tilde{\varphi}_d^{\sigma-1}}{\partial\delta}\delta+\theta^{1-\sigma}\left(\frac{\partial\kappa^x}{\partial\delta}\delta\tilde{\varphi}_{ex}^{\sigma-1}+\kappa^{ex}\frac{\partial\tilde{\varphi}_{ex}^{\sigma-1}}{\partial\delta}\delta\right)\right)$$

$$\left(\frac{\alpha}{\sigma-1}\right)\left(\xi_{M,\delta}+\Upsilon_d\xi_{\varphi_d^*,\delta}-\xi_{\varphi_{ex}^*,\delta}\left(\frac{\Upsilon_d f_d+\kappa^{ex}\Upsilon_{ex}f_{ex}}{f_d h_d^{\sigma-1}+\kappa^{ex}f_{ex,s}h_{ex}^{\sigma-1}}\right)\right)$$

$$\left(\frac{\alpha}{\sigma-1}\right)\left[\frac{\tilde{f}(1-Z_d)}{\tilde{X}}\left(\tau\delta+\Upsilon_d\xi_{\varphi_d^*,\delta}(1-\delta\tau)\right)-\xi_{\varphi_{ex}^*,\delta}\left(\frac{\Upsilon_d f_d+\kappa^{ex}\Upsilon_{ex}f_{ex}}{f_d h_{d,s}^{\sigma-1}+\kappa^{ex}f_{ex,s}h_{ex,s}^{\sigma-1}}\right)-\frac{f_{ex}}{\widetilde{fh}}\frac{\partial\kappa^{ex}}{\partial\delta}\delta\right] \tag{D.11}$$

The LHS of the FOC for τ is derived in a similar manner:

$$\left(\frac{\alpha}{\sigma-1}\right)\left[\frac{-\tau}{1-\tau}+\frac{\tilde{f}(1-Z_d)}{\tilde{X}}\left(\tau\delta+\Upsilon_d\xi_{\varphi_d^*,\tau}(1-\tau\delta)\right)-\xi_{\varphi_{ex}^*,\tau}\left(\frac{\Upsilon_d f_d+\kappa^{ex}\Upsilon_{ex}f_{ex}}{f_d h_{d,s}^{\sigma-1}+\kappa^{ex}f_{ex,s}h_{ex,s}^{\sigma-1}}\right)-\frac{f_{ex}}{\widetilde{fh}}\frac{\partial\kappa^{ex}}{\partial\delta}\tau\right] \tag{D.12}$$

Before proceeding, notice that sector average tax revenue is:

$$\bar{t}=\tau\left(u\widetilde{fh}-\tilde{f}\delta\right)$$

thus, the derivatives are:

$$\frac{\partial \bar{t}}{\partial \delta}\delta = \tau\left[\frac{-\delta\tau}{1-\tau}\widetilde{fh} + \frac{\partial \widetilde{fh}}{\partial \delta}\delta u - \tilde{f}\delta - \left(\frac{\partial \kappa^{ex}}{\partial \delta}\delta\right) f_{ex}\delta\right] \tag{D.13}$$

$$\frac{\partial \bar{t}}{\partial \tau}\tau = \tau\left[\frac{\widetilde{fh}}{1-\tau}\left(\frac{1-2\delta\tau+\delta\tau^2}{1-\tau}\right) - \delta\tilde{f} + \frac{\partial \widetilde{fh}}{\partial \tau}\tau u - \delta f_{ex}\left(\frac{\partial \kappa^{ex}}{\partial \tau}\tau\right)\right] \tag{D.14}$$

To simplify the RHS of the FOCs start with the terms:

$$\bar{t}\xi_{M,\delta} + \frac{\partial \bar{t}}{\partial \delta}\delta = \delta\tau\left[\frac{-\tilde{f}}{1-\delta\tau} - f_{ex}\left(\frac{\partial \kappa^{ex}}{\partial \delta}\delta\right)\left(1-\frac{\tilde{f}}{\widetilde{fh}}\right) + \tilde{f}\frac{\psi F_e}{\tilde{X}}\left(\Upsilon_d\xi_{\varphi_d^*,\delta} + \frac{\delta\tau}{1-\delta\tau}\right)\right] \tag{D.15}$$

(D.16)

$$\bar{t}\xi_{M,\tau} + \frac{\partial \bar{t}}{\partial \tau}\tau = \tau\left[u\widetilde{fh} + \tilde{f}\left(\frac{1-\delta-(1-\delta\tau)^2}{(1-\delta\tau)(1-\tau)}\right) - \delta f_{ex}\left(\frac{\partial \kappa^{ex}}{\partial \tau}\tau\right)\left(1-\frac{\tilde{f}}{\widetilde{fh}}\right) + \delta\tilde{f}\frac{\psi F_e}{\tilde{X}}\left(\Upsilon_d\xi_{\varphi_d^*,\tau} + \frac{\delta\tau}{1-\delta\tau}\right)\right]$$

D.1. Proof of Proposition 9

Under the assumption of Pareto distributed productivities, many of the factors in the FOCs become zero. First, from Lemma 8 $\frac{\partial \kappa}{\partial x}x = 0$ for $x = \tau, \delta$. Second, the factor:

$$\left(\Upsilon_d\xi_{\varphi_d^*,\tau} + \frac{\delta\tau}{1-\delta\tau}\right) = 0$$

To show the above, start with the solution for $\varphi_{d,s}^*$ which is straightforward once we realize that, under Pareto, $h_d = h_{ex} = \frac{k}{k-(\sigma-1)}$:

$$\varphi_d^* = \varphi_{min}\left[\tilde{f}\left(\frac{\sigma-1}{k-(\sigma-1)}\right)\frac{1-\delta\tau}{\psi F_e}\right]^{1/k} \tag{D.17}$$

While $\tilde{f}$ is a function of κ^{ex}, under Pareto the probability of export is constant. To see this:

$$\kappa = \frac{1-Z(\varphi_{ex}^*)}{1-Z(\varphi_d^*)} = \left(\frac{\varphi_d^*}{\varphi_{ex}^*}\right)^{1/k}$$

but from equation D.1:

$$\frac{\varphi_d^*}{\varphi_{ex}^*} = \left(\frac{f_d}{f_{ex}}\theta^{-1}\right)^{\frac{1}{\sigma-1}} \tag{D.18}$$

thus, κ is a function of parameters and $\tilde{f} = f_d + f_{ex}\kappa$ is not a function of the fiscal instruments. Therefore, the elasticity of the domestic (and export) productivity cutoff is equal to that of the closed economy:

$$\Upsilon_d \xi_{\varphi_d^*, x} + \frac{\delta\tau}{1-\delta\tau} = k\frac{-\delta\tau}{k(1-\delta\tau)} + \frac{\delta\tau}{1-\delta\tau} = 0 \tag{D.19}$$

Turning to the solution for the optimal tax rates. Let:

$$\tilde{k}_s = \frac{\tilde{f}_s}{\widetilde{fh}_s} = \frac{k_s}{k_s - (\sigma_s - 1)}$$

The FOC for δ becomes:

$$\left(\frac{\alpha_s}{\sigma_s - 1}\right)\left[0 - \left(\frac{-\delta_s\tau}{k_s(1-\delta_s\tau)}\right)\frac{\Upsilon_s \tilde{f}_s}{\widetilde{fh}_s} - 0\right] = -\frac{\tilde{\lambda}L\alpha_s}{\sigma_s u_s \widetilde{fh}_s}\delta\tau\left[-\frac{\tilde{f}}{1-\delta_s\tau} - 0 + 0\right]$$

$$\left(\frac{\alpha}{\sigma_s - 1}\right)\frac{\delta_s\tau}{(1-\delta_s\tau)}\tilde{k}_s^{-1} = \frac{-\tilde{\lambda}\alpha_s L}{\sigma_s u_s}\tilde{k}_s^{-1}\left(\frac{\delta_s\tau}{1-\delta_s\tau}\right)$$

$$\implies 1 - \delta_s\tau = \tilde{\lambda}\rho_s L(1-\tau) \tag{D.20}$$

the above is identical to the condition for the closed economy (equation A.18).

The FOC for τ simplifies to:

$$\sum_{s=1}^{S}\left(\frac{\alpha_s}{\sigma_s - 1}\right)\left[\frac{-\tau}{1-\tau} - \frac{(-\delta_s\tau)}{k_s(1-\delta_s\tau)}\frac{\Upsilon_{d,s}\tilde{f}_s}{\widetilde{fh}_s}\right] = -\tilde{\lambda}\sum_{s=1}^{S}\frac{\alpha_s L\tau}{\sigma_s u_s \widetilde{fh}_s}\left[u_s\widetilde{fh}_s + \tilde{f}_s\left(\frac{1-\delta_s-(1-\delta_s\tau)^2}{(1-\delta_s\tau)(1-\tau)}\right)\right]$$

$$\sum_{s=1}^{S}\left(\frac{\alpha_s}{\sigma_s - 1}\right)\left[\frac{1}{1-\tau} - \frac{\tau}{1-\delta_s\tau}\tilde{k}_s^{-1}\right] = \tilde{\lambda}L\sum_{s=1}^{S}\left(\frac{\alpha_s}{\sigma_s}\right)\left(1 + \tilde{k}_s^{-1}\left(\frac{1-\delta_s-(1-\delta_s\tau)^2}{(1-\delta_s\tau)^2}\right)\right)$$

Use the equality of D.20 and some tedious algebra to arrive at:

$$1 - \tau = \left(\sum_{s=1}^{s}\frac{\alpha_s}{k_s}\right)\left(\tilde{\lambda}L\sum_{s=1}^{S}\frac{\alpha_s\rho_s}{k_s}\right)^{-1} \tag{D.21}$$

Notice that the above is identical to the result of the closed economy (equation A.19).

Under the Pareto assumption, the S+1 equations that solve for the optimal rates in the open economy with symmetric countries are identical to those of the closed economy, therefore, the solutions to the optimal tax rates are the same.

D.2. Proof of Proposition 10

First:

$$X = \psi F_e + (1-\delta\tau)\Phi(-d) f_d < \psi F_e + (1-\delta\tau)\Phi(-d)\left(f_d + f_{ex}\kappa^{ex}\right) = \tilde{X}$$

The elasticity for the open economy ($\xi^{O}_{\varphi^*,x}$) is in equation D.9 while equation A.26 provides the formula for the closed economy counterpart. Taking the ratio:

$$\frac{\xi^{O}_{\varphi^*,x}}{\xi^{C}_{\varphi^*,x}} = \frac{X}{\tilde{X}} < 1$$

Both elasticities have the same sign (X and $\tilde{X}$ are always positive), therefore $|\xi^{O}_{\varphi^*,x}| < |\xi^{C}_{\varphi^*,x}|$.